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A note on periodic review inventory model with controllable setup cost and lead time

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Abstract

In this study, we investigate the periodic review inventory models with a mixture of backorders and lost sales by controlling lead time and setup cost simultaneously to reduce the inventory operating cost. It is assumed that the probability distribution of the protection interval, i.e., review period plus lead time, demand is unknown but its first two moments are given, we apply the minimax distribution free procedure to solve this problem. An algorithm to find the optimal solutions is developed. Specifically, from the results of numerical examples, it can be shown that, the significant savings can be achieved through the reductions of lead time and setup cost.

Scope and purpose

In most of the literature dealing with periodic review inventory problems, both lead time and setup cost are treated as constants. Recently, Ouyang and Chuang (J. Inf. Manage. Sci. 9 (1998) 25) presented a minimax distribution free procedure for the periodic review inventory model which involves a controllable lead time. We note that the paper is focusing on the benefits from lead time reduction in which setup cost is viewed as a fixed constant. From the Japanese experience of Just-In-Time (JIT) production, it has been observed in many manufacturing settings including job shops, batch shops and flow shops, whose setup cost can be reduced by investing capital. For this reason, we attempt to extend Ouyang and Chuang's model by formulating a modified periodic review model to accommodate more practical features of the real inventory systems.

Keywords: Inventory; Periodic review; Setup cost; Lead time; Minimax distribution free procedure

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1. Introduction

In traditional economic order quantity (EOQ) and economic production quantity (EPQ) models, setup cost is treaded as a constant. However, in practice, setup cost can be controlled and reduced through various efforts such as worker training, procedural changes and specialized equipment acquisition. Through the Japanese experience of using Just-In-Time (JIT) production, the advantages and benefits associated with efforts to reduce the setup cost can be clearly perceived. The ultimate goal of JIT from an inventory standpoint is to produce small lot sizes with good quality products. In order to achieve this goal, investing capital in reducing setup cost is regard as one of the effective ways. Moreover, accompanying the smaller lot sizes from lower setup cost, benefits such as greater flexibility in scheduling, lower storage space and lower investment in inventory can be obtained.

According to Silver et al. [1], the implementation of electronic data interchange (EDI) may reduce the fixed setup cost and result in new replenishment policy and the corresponding lower cost. In 1990, Nasri et al. [2] studied JIT manufacturing system, and pointed out that the impact of investing in reduced setup cost has been observed in many manufacturing settings including job shops, batch shops and flow shops. This type of investment differs from the traditional approach of investment aimed at increasing capacity because, in most production systems, production scheduling is affected directly by setup cost. In addition, setup cost control has been a topic of interest for many researchers in the field of production/inventory management. Initially, Porteus [3] introduced the concept and developed a framework of investing in reducing setup cost on the classical EOQ model. Porteus [4] extended [3] to consider the discounted effects on the EOQ model with setup cost reduction. Billington [5] considered the EPQ model without backorders and included the setup cost as a function of capital expenditure. Nasri et al. [2] investigated the effects of setup cost reduction on the EOQ model with stochastic lead time. Kim et al. [6] presented several classes of setup cost reduction functions and described a general solution procedure on the EPQ model. Paknejad et al. [7] presented a quality-adjusted lot-sizing model with stochastic demand and constant lead time, and studied the benefits of lower setup cost in the model. Sarker and Coates [8] extended EPQ model with setup cost reduction under stochastic lead time and finite number of investment possibilities to reduce setup cost.

The underlying assumption in above models is that the lead time is prescribed constant or a random variable, which therefore, is not subject to control (see, e.g. Montgomery et al. [9], Naddor [10] and Silver and Peterson [11]). In fact, as pointed out by Tersine [12], lead time usually consists of the following components: order preparation, order transit, supplier lead time, delivery time, and setup time. In many practical situations, lead time can be reduced at an added crashing cost; in other words, it is controllable. By shortening lead time, we can lower the safety stock, reduce the stockout loss, shave product costs and improve the customer service level so as to gain competitive edges in business.

The goal of JIT inventory philosophies is the focus that keeps the inventory level and lead time to a practical minimum. In 1983, Monden [13] studied Toyota production system, and clearly addressed that lead time reduction is a crux of elevating productivity. Recently, several continuous review inventory models have been developed to consider lead time as a decision variable (see, e.g. Liao and Shyu [14], Ben-Daya and Raouf [15], Ouyang et al. [16], Ouyang and Wu [17], Moon and Choi [18] and Ouyang and Chuang [19]). But in the periodic review inventory model, literature discussing lead time reduction is few. In a recent paper, Ouyang and Chuang [20] presented a minimax distribution free procedure for the periodic review inventory model which involves a controllable lead time. We note that the paper is focusing on the benefits from lead time reduction in which setup cost is treated as a fixed constant.

Though both the lead time and setup cost have been recognized as cruxes of elevating productivity, there has been little literature simultaneously examining the effects of these two factors on the inventory-control system. And hence, here we would like to investigate such an issue and extend the recent study presented by Ouyang and Chuang [20], who applied the minimax distribution free procedure to deal with the lead time reduction for the periodic review inventory models with stochastic partial backorders. That is, in this paper, instead of the fixed setup cost assumption in [20], we consider setup cost as one of the decision variables, which can be varied through capital investment. We seek to minimize the sum of capital investment cost of reducing setup cost and inventory related cost by simultaneously optimizing review period (T), setup cost (A) and lead time (L) for the periodic review model. From the numerical examples provided, we can show that the savings of total expected annual cost can be achieved by the efforts of investing in reducing setup cost. In our study, we do not require the probability distribution of the protection interval, i.e., review period plus lead time, demand to be known, however, its first and second moments are needed to be given. The purpose of this paper is to solve such a periodic review inventory model by using the minimax distribution free approach. To achieve the purpose, we develop an algorithm to find the optimal review period, optimal setup cost and optimal lead time. Moreover, two illustrative numerical examples are given in this study.

2. Notations and assumptions

To develop the proposed models, we adopt the following notations and assumptions used in Ouyang and Chuang [20] in this paper.

Notations

- D average demand per year
- h inventory holding cost per item per year
- β the fraction of the demand during the stockout period that will be backordered, $0 \leqslant \beta \leqslant 1$
- σ standard deviation of the demand per unit time
- π stockout cost per unit short
- T length of a review period
- A setup cost/setup
- L length of lead time
- X the protection interval, $T + L$, demand which has a probability density function (*p.d.f.*) f_X with finite mean $D(T + L)$ and standard deviation $\sigma\sqrt{T + L}$
- x^+ maximum value of x and 0, i.e., $x^+ = Max\{x, 0\}$.

Assumptions

1. The inventory level is reviewed every T units of time. A sufficient quantity is ordered up to the target level R, and the ordering quantity is arrived after L units of time.

2. The length of the lead time L does not exceed an inventory cycle time T so that there is never more than a single order outstanding in any cycle.
3. The target level R is the expected demand during the protection interval + safety stock (SS), and SS is the $k\times$ (standard deviation of protection interval demand), i.e., $R = D(T+L) + k\sigma\sqrt{T+L}$, where k is the safety factor and satisfies $P(X > R) = q$, q represents the allowable stockout probability during the protection interval and is given.
4. The lead time L consists of n mutually independent components. The ith component has a minimum duration a_i and normal duration b_i, and a crashing cost per unit time c_i. Further, for convenience, we rearrange c_i such that $c_1 \leqslant c_2 \leqslant \cdots \leqslant c_n$. Then, it is clear that the reduction of lead time should be first on component 1 because it has the minimum unit crashing cost, and then component 2, and so on.
5. If we let $L_0 = \sum_{j=1}^{n} b_j$ and L_i be the length of lead time with components $1, 2, \ldots, i$ crashed to their minimum duration, then L_i can be expressed as $L_i = \sum_{j=1}^{n} b_j - \sum_{j=1}^{i} (b_j - a_j),\ i = 1, 2, \ldots, n$ and the lead time crashing cost $C(L)$ per cycle for a given $L \in [L_i, L_{i-1}]$ is given by $C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.

3. Model formulation

Ouyang and Chuang [20] considered an inventory system for a periodic review model with controllable lead time, and asserted the following function of total expected annual cost which is the sum of setup cost, holding cost, stockout cost, and lead time crashing cost. Symbolically, it needs to minimize

$$EAC(T,L) = \frac{A}{T} + h\left[R - DL - \frac{DT}{2} + (1-\beta)E(X-R)^+\right] + \frac{\pi E(X-R)^+}{T} + \frac{C(L)}{T}, \tag{1}$$

where $E(X-R)^+$: the expected demand shortage at the end of cycle.

In contrast to Ouyang and Chuang's [20] model, we consider the setup cost A as a decision variable and seek to minimize the sum of the capital investment cost of reducing setup cost A and the inventory related costs (as expressed in problem (1)) by optimizing over T, A and L constrained on $0 < A \leqslant A_0$, where A_0 is the original setup cost. That is, the objective of our problem is to minimize the following total expected annual cost

$$EAC(T,A,L) = \eta M(A) + EAC(T,L), \tag{2}$$

over $A \in (0, A_0]$, where η is the fractional opportunity cost of capital per year, $M(A)$ follows a logarithmic investment function given by

$$M(A) = \frac{1}{\delta}\ln\left(\frac{A_0}{A}\right) \quad \text{for } A \in (0, A_0], \tag{3}$$

where δ is the percentage decrease in A per dollar increase in investment. This logarithmic investment function has been utilized by Nasri et al. [2], Porteus [3,4] and others [6–8].

From function (3), we note that the setup cost level $A \in (0, A_0]$. It implies that if the optimal setup cost obtained does not satisfy the restriction on A, then no setup cost reduction investment is made. For this special case, the optimal setup cost is the original setup cost.

Substitute (3) and (1) into (2) and minimize the resulting equation; we suffice to minimize

$$EAC(T,A,L) = \frac{\eta}{\delta}\ln\left(\frac{A_0}{A}\right) + \frac{A}{T} + h\left[R - DL - \frac{DT}{2} + (1-\beta)E(X-R)^+\right] + \frac{\pi E(X-R)^+}{T} + \frac{C(L)}{T}, \tag{4}$$

over $A \in (0, A_0]$.

On the other hand, since the probability distribution of the protection interval demand X is unknown, we cannot find the exact value of $E(X-R)^+$. Hence, we propose to apply the minimax distribution free procedure for our problem. Let $\mathscr{F}$ denote the class of *p.d.f.*s with finite mean $D(T+L)$ and standard deviation $\sigma\sqrt{T+L}$, then the minimax principle for this problem is to find the "most unfavorable" *p.d.f.* f_X in $\mathscr{F}$ for each (T,A,L) and then minimize over (T,A,L); more exactly, our problem is to solve

$$\underset{T,A,L}{Min}\ \underset{f_X \in \mathscr{F},}{Max}\ EAC(T,A,L), \tag{5}$$

over $A \in (0, A_0]$.

For this purpose, we need the following proposition.

Proposition 1. *For any* $f_X \in \mathscr{F}$,

$$E(X-R)^+ \leqslant \frac{1}{2}\left\{\sqrt{\sigma^2(T+L) + [R - D(T+L)]^2} - [R - D(T+L)]\right\}. \tag{6}$$

Moreover, the upper bound (6) is tight.

Proof. The proof is similar to that of Lemma 1 given by Gallego and Moon [21], and hence, we omit it.

Given that $R = D(T+L) + k\sigma\sqrt{T+L}$, and for any probability distribution of the protection interval demand X, the above inequality always holds. Then, using model (4) and inequality (6), model (5) is reduced to minimize

$$EAC(T,A,L) = \frac{\eta}{\delta}\ln\left(\frac{A_0}{A}\right) + \frac{A + C(L)}{T} + h\left[\frac{DT}{2} + k\sigma\sqrt{T+L} + \frac{1}{2}(1-\beta)\sigma\sqrt{T+L}(\sqrt{1+k^2} - k)\right] + \frac{\pi\sigma\sqrt{T+L}}{2T}(\sqrt{1+k^2} - k), \tag{7}$$

over $A \in (0, A_0]$.

In order to solve this nonlinear programming problem, we first ignore the restriction $A \in (0, A_0]$ and take the first partial derivatives of $EAC(T,A,L)$ with respect to T, A and $L \in [L_i, L_{i-1}]$, respectively.

$$\frac{\partial EAC(T,A,L)}{\partial T} = -\frac{A + C(L)}{T^2} + \frac{hD}{2} + \frac{h\sigma}{2}(T+L)^{-1/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right] + \frac{\pi\sigma}{4T}(T+L)^{-1/2}(\sqrt{1+k^2}-k) - \frac{\pi\sigma\sqrt{T+L}}{2T^2}(\sqrt{1+k^2}-k), \tag{8}$$

$$\frac{\partial EAC(T,A,L)}{\partial A} = -\frac{\eta}{\delta A} + \frac{1}{T} \tag{9}$$

and

$$\frac{\partial EAC(T,A,L)}{\partial L} = \frac{1}{2}h\sigma(T+L)^{-1/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right] + \frac{\pi\sigma}{4T}(T+L)^{-1/2}(\sqrt{1+k^2}-k) - \frac{c_i}{T}. \tag{10}$$

By examining the second-order sufficient conditions, it can be easily verified that $EAC(T,A,L)$ is not a convex function of (T,A,L). However, for fixed T and A, $EAC(T,A,L)$ is concave in $L \in [L_i, L_{i-1}]$, because

$$\frac{\partial^2 EAC(T,A,L)}{\partial L^2} = -\frac{1}{4}h\sigma(T+L)^{-3/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right] - \frac{\pi\sigma}{8T}(T+L)^{-3/2}(\sqrt{1+k^2}-k) < 0.$$

Therefore, for fixed T and A, the minimum total expected annual cost will occur at the end points of the interval $[L_i, L_{i-1}]$. On the other hand, it can be shown that, for a given value of $L \in [L_i, L_{i-1}]$, $EAC(T,A,L)$ is a convex function of (T,A) (see Appendix A for the proof). Thus, for fixed $L \in [L_i, L_{i-1}]$, the minimum value of $EAC(T,A,L)$ will occur at the point (T,A) which satisfies $\partial EAC(T,A,L)/\partial T = 0$ and $\partial EAC(T,A,L)/\partial A = 0$.

Solving above equations for T and A, respectively, produces

$$\frac{A + C(L)}{T^2} = \frac{hD}{2} + \frac{h\sigma}{2}(T+L)^{-1/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right] - \frac{\pi\sigma(T+2L)}{4T^2}(T+L)^{-1/2}(\sqrt{1+k^2}-k) \tag{11}$$

and

$$A = \frac{T\eta}{\delta}. \tag{12}$$

Substituting (12) into (11) leads to

$$\frac{T\eta/\delta + C(L)}{T^2} = \frac{hD}{2} + \frac{h\sigma}{2}(T+L)^{-1/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right] - \frac{\pi\sigma(T+2L)}{4T^2}(T+L)^{-1/2}(\sqrt{1+k^2}-k). \tag{13}$$

Theoretically, for given $\eta, \delta, h, D, \sigma, \beta, \pi, k$ (which depends on the allowable stockout probability q and the *p.d.f.* $f_X(x)$ of the protection interval demand X), and each L_i ($i = 0, 1, 2, \ldots, n$), from Eqs. (12) and (13), we can solve for (T_i, A_i, L_i); and then using model (7), we can obtain the corresponding total expected annual cost $EAC(T_i, A_i, L_i)$ for $i = 0, 1, 2, \ldots, n$. Thus, the minimum total expected annual cost can be obtained. However, in practice, since the *p.d.f.* f_X is unknown, even if the value of q is given, we cannot get the exact value of k. Therefore, in order to find the value of k, we need the following proposition.

Proposition 2. *Let X represent the protection interval demand which has a p.d.f. $f_X(x)$ with finite mean $D(T+L)$ and standard deviation $\sigma\sqrt{T+L}$, then for any real number $c > 0$,*

$$P(X > c) \leqslant \frac{\sigma^2(T+L)}{\sigma^2(T+L) + [c - D(T+L)]^2}. \tag{14}$$

Proof. See Appendix B for detail.

Because the target level $R = D(T+L) + k\sigma\sqrt{T+L}$ as mentioned earlier, if we take R instead of c in inequality (14), we get

$$P(X > R) \leqslant \frac{1}{1+k^2}. \tag{15}$$

Since it is assumed that the allowable stockout probability q during the protection interval is given, that is, $q = P(X > R)$, then from (15) we get $0 \leqslant k \leqslant \sqrt{(1/q) - 1}$.

It is easy to verify that $EAC(T, A, L)$ has a smooth curve for $k \in [0, \sqrt{(1/q) - 1}]$. Thus, we can establish the following algorithm to obtain the suitable k and hence the optimal T, A and L.

Algorithm

Step 1. For a given q, we divide the interval $[0, \sqrt{(1/q) - 1}]$ into N equal subintervals, where N is large enough. And we let $k_0 = 0$, $k_N = \sqrt{(1/q) - 1}$ and $k_j = k_{j-1} + (k_N - k_0)/N$, $j = 1, 2, \ldots, N-1$.

Step 2. For each L_i, $i = 0, 1, 2, \ldots n$, perform Steps 2-1–2-4.

Step 2-1. For given $k_j \in \{k_0, k_1, \ldots, k_N\}$, $j = 0, 1, 2, \ldots, N$, we can use a numerical search technique to compute T_{i,k_j} from Eq. (13).

Step 2-2. Substituting T_{i,k_j} into Eq. (12) determines A_{i,k_j}.

Step 2-3. Compare A_{i,k_j} and A_0.

(i) If $A_{i,k_j} \leqslant A_0$, A_{i,k_j} is feasible, then go to Step 2-4.
(ii) If $A_{i,k_j} > A_0$, A_{i,k_j} is not feasible. Take $A_{i,k_j} = A_0$ and evaluate the corresponding values of T_{i,k_j} from Eq. (13), then go to Step 2-4.

Step 2-4. Compute the corresponding total expected annual cost

$$EAC(T_{i,k_j}, A_{i,k_j}, L_i) = \frac{\eta}{\delta} \ln\left(\frac{A_0}{A_{i,k_j}}\right) + \frac{A_{i,k_j} + C(L_i)}{T_{i,k_j}}$$

$$+h\left[\frac{DT_{i,k_j}}{2}+k_j\sigma\sqrt{T_{i,k_j}+L_i}+\frac{1}{2}(1-\beta)\sigma\sqrt{T_{i,k_j}+L_i}(\sqrt{1+k_j^2}-k_j)\right]$$

$$+\frac{\pi\sigma}{2T_{i,k_j}}\sqrt{T_{i,k_j}+L_i}(\sqrt{1+k_j^2}-k_j).$$

Step 3. Find $Min_{k_j\in\{k_0,k_1,\ldots,k_N\}}EAC(T_{i,k_j},A_{i,k_j},L_i)$, and let

$$EAC(T_{i,k_{s(i)}},A_{i,k_{s(i)}},L_i)=\underset{k_j\in\{k_0,k_1,\ldots,k_N\}}{Min}EAC(T_{i,k_j},A_{i,k_j},L_i).$$

Step 4. Find $Min_{i=0,1,2,\ldots,n}EAC(T_{i,k_{s(i)}},A_{i,k_{s(i)}},L_i)$.

If $EAC(T^*,A^*,L^*)=Min_{i=0,1,2,\ldots,n}EAC(T_{i,k_{s(i)}},A_{i,k_{s(i)}},L_i)$, then (T^*,A^*,L^*) is the optimal solution; the value of $k_{s(i)}$ such that $EAC(T^*,A^*,L^*)$ exists is the optimal safety factor and we denote it by k^*. Thus, the optimal target level is $R^*=D(T^*+L^*)+k^*\sigma\sqrt{T^*+L^*}$.

4. Numerical examples

Example 1. In order to illustrate the above solution procedure, let us consider an inventory system with the following data used in Ouyang and Chuang [20]: $D=600$ units/year, $A_0=\$200$ per order, $h=\$20$/unit/year, $\sigma=7$ units/week, $\pi=\$50$, and the lead time has three components with data shown in Table 1. Besides, for setup cost reduction, we take $\eta=0.07$ and $\delta=2\times10^{-4}$.

We solve the cases when $\beta=0, 0.5, 0.8$ and 1 and $q=0.2$ (in this situation, we have $k_0=0$, $k_N=2$) and let $k_j=k_{j-1}+(k_N-k_0)/N,\ j=1,2,\ldots,N-1,\ N=200$. Applying the *Algorithm* procedure, we summarize the optimal solutions as shown in Table 2. Furthermore, to see the effects of setup cost reduction, we list the results of fixed setup cost model [20] in the same table.

From the results shown in Table 2, comparing our new model with that of fixed setup cost case, we observe the savings which range from 8.1% to 8.5%. It implies that significant savings can be easily achieved due to controlling the setup cost. Besides, from the results shown in Table 2, we see that as the value of β decreases, the smaller setup cost accompanying the larger savings of total expected annual cost are obtained.

Furthermore, if we knew the *p.d.f.* f_X, we could find an exact optimal solution for the given distribution. For example, if f_X is a normal distribution, then we can obtain the optimal (T^N,A^N,L^N) by the standard procedure and incur an minimum total expected annual cost denoted by $EAC^N(T^N,A^N,L^N)$. For fixed β and suitable k, if we use (T^*,A^*,L^*) instead of the optimal (T^N,A^N,L^N)

Table 1
Lead time data

Lead time component, i	Normal duration, b_i (days)	Minimum duration, a_i (days)	Unit crashing cost, c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Table 2
Summary of the optimal procedure solutions (T_i, L_i in week)

β	Setup cost reduction model		Fixed Setup cost model ($A = 200$)		Savings (%)
	(T^*, A^*, L^*)	$EAC(\cdot)$	(T^*, L^*)	$EAC(\cdot)$	
0.0	(7.40, 49.80, 4)	\$3829.04	(11.14, 4)	\$4184.41	8.5
0.5	(7.55, 50.82, 4)	\$3800.40	(11.29, 4)	\$4143.87	8.3
0.8	(7.63, 51.38, 4)	\$3782.79	(11.39, 4)	\$4118.86	8.2
1.0	(7.69, 51.76, 4)	\$3770.86	(11.47, 4)	\$4101.86	8.1

Note: Savings % $= \{[EAC(T^*, L^*) - EAC(T^*, A^*, L^*)]/EAC(T^*, L^*)\} \times 100\%$.

Table 3
Evaluation of EVAI

β	k^*	(T^*, A^*, L^*)	$EAC^N(\cdot)$	k^N	(T^N, A^N, L^N)	$EAC^N(\cdot)$	EVAI
0.0	1.98	(7.40, 49.80, 4)	\$2862.35	1.83	(4.52, 30.44, 4)	\$2697.08	\$165.27
0.5	1.92	(7.64, 51.43, 4)	\$2864.24	1.82	(4.54, 30.58, 4)	\$2694.35	\$169.89
0.8	1.89	(7.63, 51.38, 4)	\$2854.61	1.81	(4.56, 30.71, 4)	\$2692.68	\$161.93
1.0	1.87	(7.69, 51.76, 4)	\$2853.65	1.81	(4.56, 30.72, 4)	\$2691.54	\$162.11

Note: k^N stands for the optimal safety factor when the protection interval demand X follows a normal distribution.

for a normal distribution demand, then we can get $EAC^N(T^*, A^*, L^*)$. Hence, the added cost by using the minimax distribution free procedure instead of the normal distribution procedure is given by $EAC^N(T^*, A^*, L^*) - EAC^N(T^N, A^N, L^N)$. This is the largest amount that we would be willing to pay for the knowledge of f_X. This quantity can be regarded as the expected value of additional information (EVAI).

Example 2. Using the same data as in Example 1, we compare the procedures for the worst case distribution against the normal distribution and the results are tabulated in Table 3. For example, in the case of $\beta = 1$, we have $(T^*, A^*, L^*) = (7.69, 51.76, 4)$ and $(T^N, A^N, L^N) = (4.56, 30.72, 4)$, hence, the total expected annual cost $EAC^N(T^*, A^*, L^*) = \$2,853.65$ and $EAC^N(T^N, A^N, L^N) = \$2,691.54$, respectively. And thus, EVAI $= EAC^N(7.69, 51.76, 4) - EAC^N(4.56, 30.72, 4) = \$162.11$.

5. Concluding remarks

The primary purpose of this paper is to present a mixture of backorders and lost sales periodic review inventory model for minimizing the sum of the ordering cost, holding cost, stockout cost, and lead time crashing cost, where the review period, setup cost and lead time are considered as decision variables. In our study, we do not assume the probability distribution of the protection interval demand and apply the minimax principle to solve the problem. Numerical results show that as the value of β decreases, the smaller setup cost accompanying the larger savings of total expected annual cost could be realized.

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Appendix A.

Proof. For fixed $L \in [L_i, L_{i-1}]$, $EAC(T,A,L)$ is convex in (T,A).

For fixed $L \in [L_i, L_{i-1}]$, taking the first partial derivatives of $EAC(T,A,L)$ with respect to T and A and setting the obtaining results equal to zero results in

$$\begin{aligned} 0 &= \frac{\partial EAC(T,A,L)}{\partial T} \\ &= -\frac{A+C(L)}{T^2} + \frac{hD}{2} + \frac{h\sigma}{2}(T+L)^{-1/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right] \\ &\quad + \frac{\pi\sigma}{4T}(T+L)^{-1/2}(\sqrt{1+k^2}-k) - \frac{\pi\sigma\sqrt{T+L}}{2T^2}(\sqrt{1+k^2}-k) \end{aligned} \tag{A.1}$$

and

$$0 = \frac{\partial EAC(T,A,L)}{\partial A} = -\frac{\eta}{\delta A} + \frac{1}{T}. \tag{A.2}$$

From (A.1), we obtain

$$\begin{aligned} \frac{A+C(L)}{2T^2(T+L)} &= \frac{hD}{4(T+L)} + \frac{h\sigma}{4}(T+L)^{-3/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right] \\ &\quad + \frac{\pi\sigma}{8T}(T+L)^{-3/2}(\sqrt{1+k^2}-k) - \frac{\pi\sigma}{4T^2}(T+L)^{-1/2}(\sqrt{1+k^2}-k). \end{aligned} \tag{A.3}$$

Next, we obtain the second-order partial derivatives as follows:

$$\frac{\partial^2 EAC(T,A,L)}{\partial T^2} = \psi(T) - \frac{h\sigma}{4}(T+L)^{-3/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right], \tag{A.4}$$

where

$$\begin{aligned} \psi(T) &= \frac{2[A+C(L)]}{T^3} - \frac{\pi\sigma}{8T}(T+L)^{-3/2}(\sqrt{1+k^2}-k) \\ &\quad - \frac{\pi\sigma}{2T^2}(T+L)^{-1/2}(\sqrt{1+k^2}-k) + \frac{\pi\sigma\sqrt{T+L}}{T^3}(\sqrt{1+k^2}-k), \end{aligned}$$

$$\frac{\partial^2 EAC(T,A,L)}{\partial A^2} = \frac{\eta}{\delta A^2} = \frac{1}{TA} \quad \text{(by (A.2))} \tag{A.5}$$

and

$$\frac{\partial^2 EAC(T,A,L)}{\partial T \partial A} = \frac{\partial^2 EAC(T,A,L)}{\partial A \partial T} = -\frac{1}{T^2}. \tag{A.6}$$

In order to verify that, for fixed $L \in [L_i, L_{i-1}]$, $EAC(T,A,L)$ is convex in (T,A), we formulate the Hessian matrix H as follows:

$$H = \begin{bmatrix} \dfrac{\partial^2 EAC(T,A,L)}{\partial T^2} & \dfrac{\partial^2 EAC(T,A,L)}{\partial T \partial A} \\ \dfrac{\partial^2 EAC(T,A,L)}{\partial A \partial T} & \dfrac{\partial^2 EAC(T,A,L)}{\partial A^2} \end{bmatrix}.$$

Then, for fixed $L \in [L_i, L_{i-1}]$, we proceed by evaluating the principal minor of H at point (T,A). From (A.4), the first principal minor of H denoted by $|H_{11}|$ is

$$|H_{11}| = \frac{\partial^2 EAC(T,A,L)}{\partial T^2} = \psi(T) - \frac{h\sigma}{4}(T+L)^{-3/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right]. \tag{A.7}$$

Further, we let

$$\wp(T) = \frac{A + C(L)}{2T^2(T+L)} - \frac{\pi\sigma}{8T}(T+L)^{-3/2}(\sqrt{1+k^2}-k) + \frac{\pi\sigma}{4T^2}(T+L)^{-1/2}(\sqrt{1+k^2}-k), \tag{A.8}$$

then, from (A.3), it implies

$$\wp(T) > \frac{h\sigma}{4}(T+L)^{-3/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-k)\right]. \tag{A.9}$$

Therefore, from (A.7) and (A.9), we get

$$\begin{aligned} |H_{11}| &> \psi(T) - \wp(T) \\ &= \frac{[A + C(L)](3T+4L)}{2T^3(T+L)} + \frac{\pi\sigma(T+4L)}{4T^3}(T+L)^{-1/2}(\sqrt{1+k^2}-k) > 0. \end{aligned} \tag{A.10}$$

From (A.4)–(A.6), we obtain the second principal minor of H denoted by $|H_{22}|$ as

$$\begin{aligned} |H_{22}| &= \left\{\psi(T) - \frac{h\sigma}{4}(T+L)^{-3/2}\left[k + \frac{1}{2}(1-\beta)(\sqrt{1+k^2}-1)\right]\right\}\frac{1}{TA} - \frac{1}{T^4} \\ &> \left\{\frac{[A+C(L)](3T+4L)}{2T^3(T+L)} + \frac{\pi\sigma(T+4L)}{4T^3}(T+L)^{-1/2}(\sqrt{1+k^2}-k)\right\}\frac{1}{TA} \\ &\quad -\frac{1}{T^4} \quad \text{(by (A.10))} \\ &> \left\{\frac{3[A+C(L)]}{2T^3}\right\}\frac{1}{TA} - \frac{1}{T^4} \\ &> \left[\frac{A}{T^3} + \frac{C(L)}{T^3}\right]\frac{1}{TA} - \frac{1}{T^4} \\ &= \frac{C(L)}{T^4 A} > 0. \end{aligned}$$

Therefore, it is clear to see that, for fixed $L \in [L_i, L_{i-1}]$, $EAC(T,A,L)$ is convex in (T,A). □

Appendix B.

Proof. For any real number $a(\neq -c)$, we have

$$\begin{aligned} P(X > c) &= P(X + a > c + a) \\ &\leqslant P[(X + a)^2 > (c + a)^2] \\ &\leqslant \frac{E(X + a)^2}{(c + a)^2} \quad \text{(by Morkov inequality)} \\ &\leqslant \frac{\sigma^2(T + L) + [D(T + L) + a]^2}{(c + a)^2}. \end{aligned} \tag{B.1}$$

The right-hand side of the last equality has a minimum value when

$$a = \frac{\sigma^2(T + L)}{c - D(T + L)} - D(T + L).$$

Because (B.1) holds for any $a(\neq -c)$, hence, if we put $a = (\sigma^2(T + L)/c - D(T + L)) - D(T + L)$ into (B.1), we get

$$P(X > c) \leqslant \frac{\sigma^2(T + L)}{\sigma^2(T + L) + [c - D(T + L)]^2}.$$

The proof is completed. □

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