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The analysis of optimal control model in matching problem between manufacturing and marketing

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Abstract

A mathematical model is developed and presented to solve the optimal control problem in matching between manufacturing and marketing when the market demand possesses the property of linearity. The main purpose of this paper is to investigate the optimal choice between production rate which may affect the inventory level and the optimal sales rate at each time through different pricing strategies so as to achieve the maximum profit for a given planning horizon. The results demonstrated that the relative size of the output yielded by minimizing average cost and yielded by maximizing profit at constant-zero inventory can determine the optimal production rate and sales rate. Furthermore, the optimal inventory policy can be determined to be a constant-zero inventory policy or constant-positive inventory policy, or mixed inventory policy, according to the value of optimal production rate.

Keywords: Production; Pricing; Optimization; Decision analysis

1. Introduction

The rapid changes in business environment and the popularity of information system already modify the consumers' behaviors and yield a huge impact on business strategies. In the era of consumers' sovereign, the firms that can satisfy the consumers' requirements will survive and become a winner in the market. The market trend empowers the consumers to determine product design and choose what products they need. This trend reflects more on the roles of logistics sector which coordinates manufacturers, wholesalers, and retailers than the consumers, and connects the concept gap between manufacturers and consumers.

The economic trend makes the logistics industry more liberal than ever before. The introduction of point-on-sale (POS) [11] information system speeds up the information generation regarding the market

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supply, demand and competition, and also enhances the operating efficiency of the supply chain to increase the total benefits. The POS system can link computer systems which can promptly identify products and prices by means of coding system with auto-tellers to generate the sales information by computing, analyzing, and reporting on the spot and release these information to the users [1]. Facing the rapid change in business operating patterns, the manager should consider not only the manufacturing strategies but also the sales policies. In other words, the firm needs to combine the manufacturing decision variables of the optimal production rate to minimize total cost, and the sales decision variables of the optimal selling price to maximize total profits.

Many researchers already focused on the issue about the matching problem between manufacturing and marketing. Pekelman [10] presented a dynamical production-inventory model in which he treated pricing as a decision variable in a given time interval. Feichtinger and Hartl [4] focused on the same problem and attempted to find out the optimal matching between optimal pricing and production rate by releasing of some constraints when out of stock is permitted. In addition, Jorgensen [6], Eliashberg and Steinberg [3], and Gaimon [5] incorporated the other factors such as sales terms, production scale into the production-inventory model to explore the optimal matching strategy. However, all these researchers neglected the important effect of the variation of time interval on the strategies, but considered the matching between pricing and production rate at a given time interval. Recently, some researchers such as Lee and Kim [9], Kim and Lee [7,8] attempted to find out the optimal order quantity and selling price to maximize profits for sellers who need to order commodity from the suppliers. In these previous studies, the researchers had not discussed the effect of pricing at each time interval on production rate. This study attempts to develop a matching model between manufacturing and marketing. The model are constructed under the assumption that the channel of supply and demand has been constructed and the effects induced by the factors, including the quality of product, service attitude and the capacity of the production, are considered through the demand function and the cost function of making products. We constructed a mathematical model to analyze the production rate and the control of the sales rate at each time through different pricing strategies, in order to achieve the optimal profit within the planning horizon. The constructed model is suitable for the novel products which sales rate can be controlled through price strategy, such as mobile phone, personal digital assistant, etc. The properties of the optimal solution derived from this model can serve as some decision guide-lines to determine the optimal production rate based on the information about market situation received from the daily information-communication system and to set up the optimal inventory policy simultaneously.

2. Notations

The following notations are used in constructing our model.

Parameters and given functions

$[0, T]$: Allowable horizon for future decision making.

h: Unit inventory holding cost per unit time.

$P_t = b - aS_t$: Demand function at t, $t \in [0, T]$, where P_t and S_t denote the price and sales rate at t, respectively. a and b denote the increasing of the reducing sale price per unit product and price ceiling, respectively. $a > 0$, $b > 0$ and $0 \leqslant S_t \leqslant b/a$.

$c(q)$: The production cost per unit time with production rate q. $c(0) \geqslant 0$, $c'(q) > 0$ and $c''(q) > 0$, the production cost includes both fixed and variable costs.

q_{ACM}: The output yielded by minimizing average cost, and $q_{\mathrm{ACM}} \geqslant 0$, i.e. $c(q_{\mathrm{ACM}})/q_{\mathrm{ACM}} = \min_q c(q)/q$ and $qc'(q) - c(q) \geqslant 0 \Longleftrightarrow q \geqslant q_{\mathrm{ACM}}$.

q_{PM}: The output yielded by maximizing profit at constant-zero inventory, and $q_{\mathrm{PM}} \geqslant 0$, i.e. q_{PM} is the optimal solution of $\max_q[(b - aq)q - c(q)]$.

Decision variables and functions

$y(t)$: Sale quantities within $[0, t]$. $0 \leqslant t \leqslant \bar{t}_y \leqslant T$ and $\bar{t}_y$ represents the terminal point of sales horizon corresponding to y, $y(0) = 0$ and $y(t) \leqslant qt\ \forall t \in [0, \bar{t}_y]$.

q: Production rate.

$y(\bar{t}_y)/q$: The terminal point of production horizon corresponding to y. $[0, y(\bar{t}_y)/q]$ represents the production horizon and $(y(\bar{t}_y)/q) \leqslant T$.

t_y: The maximum t that satisfied $y(t) = qt$ and $t_y \in [0, T]$. Since $y(0) = 0$, there must exist a t_y. $[0, t_y]$ represents zero inventory horizon corresponding to y.

3. Mathematical model

Fig. 1 illustrates the matching problem between manufacturing and marketing, where the products are manufactured with production rate q and the products are assumed to be sold accordingly to function $y(t)$, which represents the accumulated sale quantities within $[0, t]$. The sales rate $y'(t)$ is assumed to be equal to the production rate within $[0, t_y]$, and is lower than the production rate for $t > t_y$.

In the present study, we supposed that the decision-maker presumes the market will remain stable within the planning horizon $[0, T]$ and the changes in the supply of the production factors, the market preferences, and the rivalry's responses are minimal and can be neglected.

Assuming that the decision-maker can choose constant-zero inventory policy in the planning horizon. The unit profit during the zero-inventory horizon is $(b - aq)q - c(q)$, since $(b - aq)q - c(q)$ is a non-negative real number, it implies that q will fall in the following sets:

$$\{q|(b - aq)q - c(q) \geqslant 0\} = \{q|q_s \leqslant q \leqslant q_l\}$$

where q_s and q_l represent the roots of $(b - aq)q - c(q) = 0$ (Note: since $c'(q) > 0$ and $c''(q) > 0$, thus $(b - aq)q - c(q)$ possesses at most two roots).

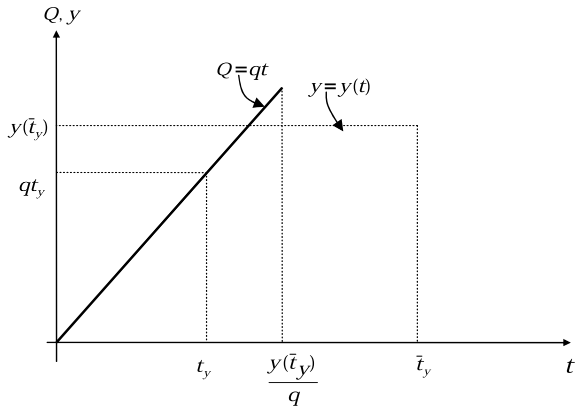

Fig. 1. The relationship betweem accumulative throughputs $Q = qt$ and sales quantities $y = y(t)$.

By using the inequality $(b - aq)q \geqslant c(q)$ and Fig. 1, it is proved that in order to maximize the profit, $\bar{t}_y$ must be equal to T (cf. Chen and Chu [2]).

The purpose of this study is to investigate how to determine the production rate q and how to control the sales rate $y'(t)$ at each time t through different pricing strategies, in order to achieve the optimal profit within the time horizon $[0, T]$. The corresponding mathematical model for the matching problem between manufacturing and marketing can be expressed as the following:

Model I:

$$\begin{aligned}\underset{(q,y)}{\mathrm{Max}}\ J(q,y) = \underset{q}{\mathrm{Max}}\ \underset{y}{\mathrm{Max}}&\left[\int_0^{t_y} (-aq+b)q\,\mathrm{d}t + \int_{t_y}^{T} (-ay'(t)+b)y'(t)\,\mathrm{d}t\right] - \left[\int_0^{y(T)/q} c(q)\,\mathrm{d}t\right]\\ &- \left[h\int_{t_y}^{y(T)/q} (qt - y(t))\,\mathrm{d}t + \int_{y(t)/q}^{T} (y(T)-y(t))\,\mathrm{d}t\right] \qquad (1)\end{aligned}$$

$$\begin{aligned}\text{s.t.}\quad & y(t_y) = qt_y \quad \text{and} \quad y(t) < qt, \ \ \forall t \in (t_y, T],\\ & 0 \leqslant y'(t) \leqslant \frac{b}{a}, \quad \forall t \in [t_y, T],\\ & q_s \leqslant q \leqslant q_l,\end{aligned}$$

where $\int_0^{t_y} (-aq+b)q\,\mathrm{d}t$ evaluates the revenue within $[0, t_y]$ in which the production rate is equal to the sales rate. $\int_{t_y}^{T} (-ay'(t)+b)y'(t)\,\mathrm{d}t$ evaluates the revenue within $[t_y, T]$ in which the production rate is not equal to the sales rate. $\int_0^{y(T)/q} c(q)\,\mathrm{d}t$ evaluates the total production cost within production horizon. $h\int_{t_y}^{y(T)/q} (qt - y(t))\,\mathrm{d}t$ evaluates the inventory holding cost within production horizon. $h\int_{y(T)/q}^{\mathrm{T}} (y(T) - y(t))\,\mathrm{d}t$ evaluates the inventory holding cost within $[y(T)/q, T]$.

Model I can be summarized as follows:

Model I′:

$$\begin{aligned}\underset{(q,y)}{\mathrm{Max}}\ J(q,y) = \underset{q}{\mathrm{Max}}\ \underset{y}{\mathrm{Max}}&\int_{t_y}^{T} (-ay'^2(t) + by'(t) + hy(t))\,\mathrm{d}t + (b-aq)qt_y - \frac{c(q)\mathrm{y}(T)}{q} + \frac{hqt_y^2}{2}\\ &+ \frac{hy^2(T)}{2q} - hTy(T) \qquad (2)\end{aligned}$$

$$\begin{aligned}\text{s.t.}\quad & y(t_y) = qt_y \quad \text{and} \quad y(t) < qt, \ \ \forall t \in (t_y, T],\\ & 0 \leqslant y'(t) \leqslant \frac{b}{a}, \quad \forall t \in [t_y, T],\\ & q_s \leqslant q \leqslant q_l.\end{aligned}$$

4. The optimal solution

Let y_q be the optimal solution of $\max_y J(q, y)$ under a given production rate q, thus, according to the results in Chen and Chu [2], y_q can be summarized as follows:

$$y_q(t) = \begin{cases} qt \quad \forall t \in [0,T] \quad \text{if } q - \sqrt{\dfrac{(b-aq)q-c(q)}{a}} \leqslant 0 \\ \qquad \text{(in this case } t_{y_q} = T;\ \text{constant-zero inventory policy)}, \\ \begin{cases} qt \quad \forall t \in [0, t_{y_q}] \\ -\dfrac{h}{4a}(t-t_{y_q})^2 + qt \quad \forall t \in [t_{y_q}, T] \end{cases} \quad \text{if } 0 < q - \sqrt{\dfrac{(b-aq)q-c(q)}{a}} < \dfrac{hT}{2a} \\ \qquad \left(\text{in this case } t_{y_q} = T - \dfrac{2a}{h}\left(q - \sqrt{\dfrac{(b-aq)q-c(q)}{q}}\right);\ \text{mixed inventory policy}\right), \\ -\dfrac{h}{4a}t^2 + \dfrac{1}{4a}\left[2aq + hT + 4a\dfrac{(b-aq)q-c(q)}{2aq-hT}\right]t \quad \forall t \in [0,T] \quad \text{if } q - \sqrt{\dfrac{(b-aq)q-c(q)}{a}} \geqslant \dfrac{hT}{2a} \\ \qquad \text{(in this case } t_{y_q} = 0;\ \text{constant-positive inventory policy)}. \end{cases} \tag{3}$$

Therefore, the Model I′ can be expressed as

Model I″

$$\max_{q_s \leqslant q \leqslant q_l} J(q, y_q).$$

It is known that the necessary and sufficient condition for q^* being the optimal solution of Model I″ is that (q^*, y_{q^*}) is the optimal solution of Model I. For simplicity, let $J(q) = J(q, y_q)$, according to Eq. (3), $J(q)$ can be expressed as

$$J(q) = \begin{cases} J_1(q) \quad \text{if } q \in S_1 \\ \qquad \text{where} \begin{cases} J_1(q) = [(b-aq)q - c(q)]T, \\ S_1 = \left\{ q | q_s \leqslant q \leqslant q_1 \quad \text{and } a - \dfrac{\sqrt{(b-aq)q-c(q)}}{a} \leqslant 0 \right\}, \end{cases} \\ J_2(q) \quad \text{if } q \in S_2 \\ \qquad \text{where} \begin{cases} J_2(q) = [(b-aq)q-c(q)]T \\ \qquad + [(b-aq)q-c(q)]\left[\dfrac{4\sqrt{a}}{3h}\sqrt{(b-aq)q-c(q)} - \dfrac{(b-aq)q-c(q)}{2hq} - \dfrac{aq}{h}\right] + \dfrac{a^2q^3}{6h}, \\ S_2 = \left\{ q | q_s \leqslant q \leqslant q_l \quad \text{and } q - \sqrt{\dfrac{(b-aq)q-c(q)}{q}} \leqslant \dfrac{hT}{2a} \right\}, \end{cases} \\ J_3(q) \quad \text{if } q \in S_3 \\ \qquad \text{where} \begin{cases} J_3(q) = \dfrac{(2b - 2c(q) - hqT)^2 T}{8q(2aq - hT)} + \dfrac{h^2T^3}{48a}, \\ S_3 = \left\{ q | q_s \leqslant q \leqslant q_t \quad \text{and } q - \sqrt{\dfrac{(b-aq)q-c(q)}{a}} \geqslant \dfrac{hT}{2a} \right\}. \end{cases} \end{cases} \tag{4}$$

In Appendix A, it is proved that $J_3(q) \leqslant J_3(\bar{q}) = J_2(\bar{q})$, where $\bar{q} = \min_q S_3$ ($\bar{q}$ refers to as production capacity or upper limit of production). It implies that the optimal production rate can be determined by just comparing the objective value of $J_1(q)$, $q \in S_1$ and $J_2(q)$, $q \in S_2$, hence $\max_{q \in S_1 \cup S_2 \cup S_3} J(q) = \max_{q \in S_1 \cup S_2} J(q)$.

Based on the characteristic of Model I″ and expression of $J(q)$, we found that the optimal solutions can be determined graphically through the discussion of relationship between $J(q)$, $y = aq^2T$ and

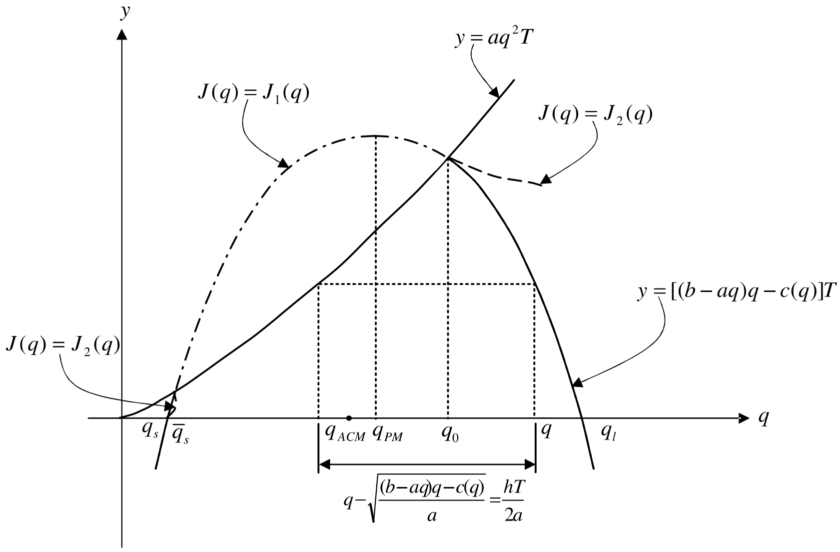

Fig. 2. Case 1: linkage among $y = aq^2T$, $J(q)$ and $y = [(b - aq)q - c(q)]T$.

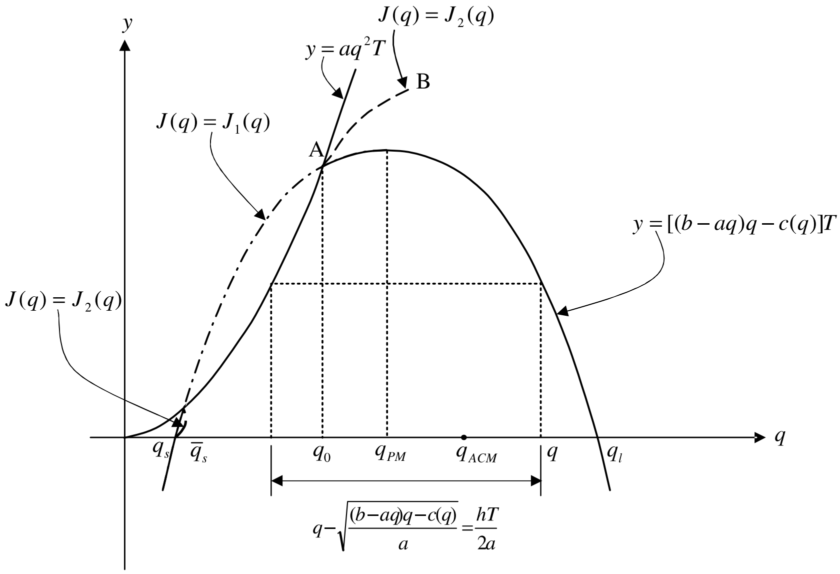

Fig. 3. Case 2: linkage among $y = aq^2T$, $J(q)$ and $y = [(b - aq)q - c(q)]T$.

$y = [(b - aq)q - c(q)]T$. According to the properties in Appendix B, we summarized that there exists three possible relations between $J(q)$, $y = aq^2T$ and $y = [(b - aq)q - c(q)]T$ as shown in Figs. 2–4.

Denote q_0 as the largest lateral-coordinate intersection of $y = aq^2T$ and $y = [(b - aq)q - c(q)]T$. In the case of Fig. 2, it is shown that there exists $q \in [q_s, q_l]$, such that $aq^2T \leqslant [(b - aq)q - c(q)]T$, q_0 locates at the right hand side of q_{PM}, and we have $S_1 = [\bar{q}_s, q_0]$, $S_2 = [q_{s,}\bar{q}_s] \cup [q_0, \bar{q}]$, where $\bar{q}$ satisfies $\bar{q} - ((b - a\bar{q})\bar{q} - c(\bar{q})/\bar{q})^{1/2} = hT/2a$.

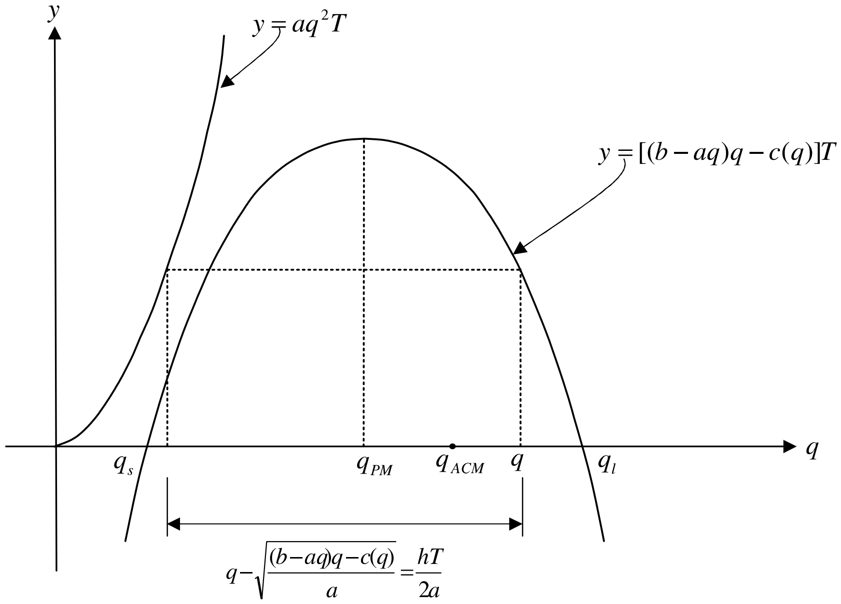

Fig. 4. Case 3: linkage among $y = aq^2T$ and $y = [(b-aq)q - c(q)]T$.

It is obvious that

$$\max_{q \in S_1 \cup S_2} J(q) = J(q_{\mathrm{PM}}) = J(q^*).$$

In the case of Fig. 3, it is shown that there exists $q \in [q_s, q_l]$, such that $aq^2T \leqslant [(b-aq)q - c(q)]T$, q_0 locates at the left-hand side of q_{PM} and $S_1 = [\bar{q}_s, q_0]$, $S_2 = [q_s, \bar{q}_s] \cup [q_0, \bar{q}]$. Thus, we have

$$\max_{q \in S_1 \cup S_2} J(q) = \max_{q \in S_2} J_2(q).$$

Since $J_2'(q) > 0$, $\forall q < q_{\mathrm{PM}} \Theta \bar{q}$ and $J_2'(q) < 0$, $\forall q > q_{\mathrm{ACM}} \Theta \bar{q}$ (c.f. Appendix B), it is shown that

$$\text{if } q_{\mathrm{PM}} < \bar{q} \quad \text{then } \max_{q \in S_2} J_2(q) = \max_{q \in [q_{\mathrm{PM}}, \bar{q} \Theta q_{\mathrm{ACM}}]} J_2(q),$$

$$\text{if } q_{\mathrm{PM}} \geqslant \bar{q} \quad \text{then } \max_{q \in S_2} J_2(q) = J_2(\bar{q}).$$

In the case of Fig. 4, q_0 does not exist, hence $S_1 = \phi$, $S_2 = [q_s, \bar{q}]$, thus, the optimal solution is determined only by $J_2(q)$, i.e.

$$\max_{q \in S_1 \cup S_2} J(q) = \max_{q \in S_2} J_2(q).$$

Following the same discussion as in the case of Fig. 3, we have

$$\text{if } q_{\mathrm{PM}} < \bar{q} \quad \text{then } \max_{q \in S_2} J_2(q) = \max_{q \in [q_{\mathrm{PM}}, \bar{q} \Theta q_{\mathrm{ACM}}]} J_2(q),$$

$$\text{if } q_{\mathrm{PM}} \geqslant \bar{q} \quad \text{then } \max_{q \in S_2} J_2(q) = J_2(\bar{q}).$$

From the above, the optimal solution of production rate q^* can be summarized as follows:
Case I: if $q_{\mathrm{ACM}} \leqslant q_{\mathrm{PM}}$ then $q - (((b-aq)q - c(q))/a)^{1/2} \leqslant 0$ and $q^* = q_{\mathrm{PM}}$.
Case II: if $q_{\mathrm{ACM}} > q_{\mathrm{PM}}$ and $q_{\mathrm{PM}} \geqslant \bar{q}$

then $q - (((b - aq)q - c(q))/a)^{1/2} \geqslant hT/2a$ and $q^* = \bar{q}$.

Case III: if $q_{\mathrm{ACM}} > q_{\mathrm{PM}}$ and $q_{\mathrm{PM}} < \bar{q}$ then $0 < q - (((b - aq)q - c(q))/a)^{1/2} < hT/2a$ and $q^* \in [q_{\mathrm{PM}}, \bar{q}\Theta q_{\mathrm{ACM}}]$.

5. Discussions and conclusion

Accompanied by the rapid change of business environment such as popularity of information system and the decrease in information cost, a firm needs to devote all efforts to find out the optimal strategy in facing the changing environment. The quick response to the environmental threat plays a very important factor to survive. This study can help the decision-maker to achieve the objective, since our model offers a benefit for a firm to decide the production rate and selling price simultaneously to maximize the profits. The decision maker can easily figure out the optimal pattern of inventory policy by knowing the relative size of the output yielded by minimizing average cost, and the output yielded by maximizing profit at constant-zero inventory. In the present study, we obtain the following conclusions:

(1) If the output at minimized average cost is less than or equal to the output of maximized profit at constant-zero inventory, the optimal production rate shall be equal to the output of maximized profit at constant-zero inventory. The corresponding optimal inventory policy is constant-zero inventory policy.
(2) If the output at minimized average cost is greater than the output of maximized profit at constant-zero inventory, and the output of maximized profit at constant-zero inventory is greater than or equal to the upper limit of production rate, then the optimal production rate is equal to the upper limit of production rate. The corresponding optimal inventory policy is constant-positive inventory policy.
(3) If the output at minimized average cost is greater than the output of maximized profit at constant-zero inventory, and the output of maximized profit at constant-zero inventory is less than the upper limit of production rate, then the optimal production rate falls between the output of maximized constant-zero inventory profit and the minimized value of upper limit of production rate and the output at minimized average cost. According to the value of optimal production, the corresponding optimal inventory policy can be determined to be either constant-positive inventory, constant-zero inventory or mixed inventory policy.

To fully implement the matching between manufacturing and marketing, a decision-maker must keep closed control over the information in association with market supplies, demands and market situations. This model serves as an automatic mechanism to support a quick-response system so that the production information is released to manufacturing unit and the sales information can support the decision-maker to determine the optimal inventory policy and the sensitivity analysis can be obtained through the mechanism.

Appendix A

Prove that $J_3(q) \leqslant J_3(\bar{q}) = J_2(\bar{q})$, where $\bar{q} = \min_q S_3$.

By Eq. (3), we know that $y_q'(0) = (bq - c(q) - (h^2T^2/4a))/2aq - hT$ and $q \in S_3$. This implies that

$$y_q'(0) \leqslant q. \tag{A.1}$$

By using Eq. (A.1) and Fig. 5, the total inventory holding costs of $J(q, y_q)$ are more than $J(y_q'(0), y_q)$. This means that

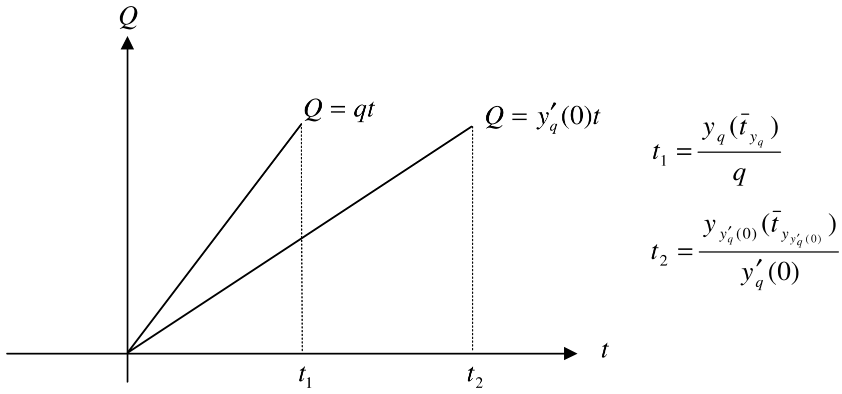

Fig. 5. The relationship betweem accumulative throughputs $Q = qt$ and sales quantities $Q = y'_q(0)t$.

$$J(q, y_q) < J(y'_q(0), y_q) \leqslant J(y'(0), y_{y'(0)}) = J(\bar{q}, y_{\bar{q}}), \quad \bar{q} = \min_q S_3.$$

Hence

$$J_3(q) \leqslant J_3(\bar{q}) = J_2(\bar{q}). \tag{A.2}$$

Appendix B

B.1. Determine the relation between q_{PM}, q_0 and q_{ACM}

We assume that the curve $y = aq^2T$ and the curve $J_1(q) = [(b - aq)q - c(q)]T$ have the intersection points, let q_0 be the intersection with maximum lateral coordinate, therefore, we have the following property:

Property 1
(a) *if* $q_PM < q_ACM$ *then* $q_0 < q_PM$;
(b) *if* $q_PM > q_ACM$ *then* $q_0 > q_PM$.

Proof 1

$$\begin{aligned}(J_1(q) - aq^2T)|_{q=q_{\mathrm{PM}}} &= [(b - 2aq)q - c(q)]T|_{q=q_{\mathrm{PM}}} = [(b - 2aq)q - c'(q) - c'(q)q - c(q)]T|_{q=q_{\mathrm{PM}}} \\ &= [c'(q_{\mathrm{PM}})q_{\mathrm{PM}} - c(q_{\mathrm{PM}})]T \text{ (by definition of } q_{\mathrm{ACM}} \text{ and } q_{\mathrm{PM}})\end{aligned}$$

therefore, we have

$$(J_1(q) - aq^2T)|_{q=q_{\mathrm{PM}}} \begin{cases} > 0 & \text{if } q_{\mathrm{PM}} > q_{\mathrm{ACM}}, \\ = 0 & \text{if } q_{\mathrm{PM}} = q_{\mathrm{ACM}}, \\ < 0 & \text{if } q_{\mathrm{PM}} < q_{\mathrm{ACM}}. \end{cases}$$

Hence, we get the property that q_{PM} is between q_{ACM} and q_0.

B.2. Find the derivative of $J_1(q)$ and $J_2(q)$

Since $J_1(q) = [(b - aq)q - c(q)]T, \forall q \in S_1$ therefore, $J'_1(q) = (b - 2aq - c'(q))T$. By definition of q_{PM}, we have $J'_1(q) > 0, \forall q < q_{\mathrm{PM}}$ and $J'_1(q) < 0, \forall q > q_{\mathrm{PM}}$,

$$J_2(q) = [(b-aq)q - c(q)]\left[T + \frac{4\sqrt{a}}{3h}\sqrt{(b-aq)q-c(q)} - \frac{(b-aq)q-c(q)}{2hq} - \frac{aq}{h}\right] + \frac{a^2q^3}{6h}, \quad \forall q \in S_2.$$

Let $\chi(q) = (b-aq)q - c(q)$ and $\chi'(q) = b - 2aq - c'(q)$.

Hence

$$J_2'(q) = \chi\left[T + \frac{4\sqrt{a}}{3h}\sqrt{\chi} - \frac{\chi}{2hq} - \frac{aq}{h}\right] + \frac{a^2q^3}{6h},$$

and then

$$hJ_2'(q) = [b-2aq-c'(q)]\left[2aq - b + \frac{c(q)}{q} + t_{y_q}h\right] + \frac{1}{2}\left[aq - \frac{\chi}{q}\right]^2$$

where $t_{y_q}h = Th - 2aq + 2\sqrt{a}\sqrt{\chi}$ (recalling Eq. (3)).

Since $J_1'(q)/T = b - 2aq - c'(q) > 0$, $\forall q < q_{\mathrm{PM}}$ and $2aq - b + c(q)/q \geqslant 0$, $\forall q \in S_2$.

Hence $J_2'(q) > 0$, $\forall q < q_{\mathrm{PM}}\Theta\bar{q}$ (define that $q_{\mathrm{PM}}\Theta\bar{q} \equiv \min\{q_{\mathrm{PM}}, \bar{q}\}$) $J_2'(q)$ can also be expressed as

$$\begin{aligned} hJ_2'(q) &= [b-2aq-c'(q)]\left[2aq - b + \frac{c(q)}{q} + t_{y_q}h\right] + \frac{1}{2}\left[aq - \frac{\chi}{q}\right]^2 \\ &= [b-2aq-c'(q)]\left[-\left(b - 2aq - \frac{c(q)}{q}\right) + t_{y_q}h\right] + \frac{1}{2}\left[b - 2aq - \frac{c(q)}{q}\right]\left[b - 2aq - \frac{c'(q)}{q}\right]. \end{aligned}$$

By definition of q_{ACM}, we have

$$hJ_2'(q) < [b-2aq-c'(q)]\left[-\left(b - 2aq - \frac{c(q)}{q}\right) + t_{y_q}h\right] + \frac{1}{2}\left[b - 2aq - \frac{c(q)}{q}\right][b - 2aq - c'(q)],$$

$$hJ_2'(q) < [b-2aq-c'(q)]\left[-\frac{1}{2}\left(b - 2aq - \frac{c(q)}{q}\right) + t_{y_q}h\right] < 0, \quad \forall q > q_{\mathrm{ACM}}.$$

Hence $J_2'(q) < 0$, $\forall q > q_{\mathrm{ACM}}\Theta\bar{q}$.

From the above discussion, we have the following properties:

Property 2. *$J_1'(q) > 0$, $\forall q < q_{\mathrm{PM}}$ and $J_2'(q) < 0$, $\forall q > q_{\mathrm{PM}}$.*

Property 3. *$J_2'(q) > 0$, $\forall q < q_{\mathrm{PM}}\Theta\bar{q}$ and $J_2'(q) < 0$, $\forall q > q_{\mathrm{ACM}}\Theta\bar{q}$.*

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