

行政院國家科學委員會專題研究計畫 成果報告

情境調整因子之模糊時間序列方法 研究成果報告(精簡版)

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計畫主持人：王佳文

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處理方式：本計畫涉及專利或其他智慧財產權，2年後可公開查詢

中華民國 98 年 10 月 30 日

行政院國家科學委員會補助專題研究計畫 成果報告 期中進度報告

情境調整因子之模糊時間序列方法

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成果報告類型(依經費核定清單規定繳交)： 精簡報告 完整報告

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國際合作研究計畫國外研究報告書一份

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查詢

執行單位：南華大學電子商務系

中華民國 98 年 9 月 31 日

行政院國家科學委員會專題研究計畫成果報告

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中文摘要

本計畫提出一個多期多屬性模糊時間序列架構，其中結合分群方法與循序權重平均方法。本計畫改良 Cheng 等人(2008) 所提模糊叢集時間序列的概念，結合指數平滑預測法之概念，加入誤差修正因子，其利用(1) 指數平滑預測法與(2) 最大熵循序加權運算子(ME-OWA operator)來調整誤差。本計畫所提出的架構可解決以下三個問題 (1)主觀決定字集分布與語意區間長，(2) 單一屬性預測而非多屬性。再者，我們可以根據不同情境 α 去調整多期的預測值。為了驗證我們所提出的架構，我們使用阿拉巴馬新生人數、台灣期貨交易所、與台灣股票交易所之資料集當作實驗資料。最後計畫結果與 Huang 和 Cheng 學者比較，所得預測正確率優於所比較的方法。

關鍵字：模糊時間序列，循序權重平均運算子，模糊分群，模糊邏輯關係

FUZZY TIME SERIES METHOD BASED ON SITUATION ADJUSTMENT FACTOR

Abstract

This project proposes a high-order multiple attribute fuzzy time series model, which incorporates a clustering method and order weighted averaging (OWA) operator. This research improved fuzzy clustering time series concept by Cheng et al.'s [2], further, this project combine the Exponential Smoothing method and adjust error method. We use exponential Smoothing forecasting and ME-OWA operator to adjust error. The proposed model can deal with the fact that there is: (1) a persuasiveness lacking in determining the universe of discourse and the linguistic length of intervals, and (2) only one attribute that is usually considered in forecasting, not multiple attributes. Furthermore, we can accord a different situation α to adjust high order forecasting values. To verify the proposed model, we use the yearly data on enrollments at the University of Alabama, TAIFEX (Taiwan Futures Exchange), and TAIEX (Taiwan Stock Exchange Capitalization Weighted Stock Index) as experimental datasets. Finally, this project compares forecasting performances of the proposed methods with Huang et al.'s [1] and Cheng et al.'s [2] models; the results of empirical analysis conclude that the proposed model surpasses the listing models in accuracy.

Keywords: Fuzzy time series, Order Weighted Averaging (OWA) operator, Fuzzy clustering, Fuzzy logic relationship (FLR)

1. Introduction

In recent years, time series models have utilized the fuzzy theory to solve various forecasting problems, such as university enrolment forecasting [3], financial forecasting [1, 4], and temperature forecasting [5]. Many researchers have proposed different methods to predict stock price based on fuzzy time series. In 1993, Song and Chissom [6] proposed the first fuzzy time series method, where the definitions, the time-invariant model [4], and the time-variant model [1] of fuzzy time series were presented. The following research proposed simple calculation methods to get a higher forecasting accuracy [3]. Huang and Yu utilized two methods, distribution-based and average-based length, to set the linguistic intervals of the universe of discourse to improve forecasting accuracy [7]. From the literature above, there are drawbacks: (1) Many factors in stock markets, such as financial report, macroeconomic data, and the fluctuations of foreign stock markets, influence the decisions of stock investors practically, and therefore, multiple attributes should be considered in forecasting models. (2) In stock market forecasting, it is not reasonable to partition the universe of the discourse for the dataset, because the distribution of stock prices is not uniform.

Therefore, this project proposes a multiple attribute fuzzy time series model, which incorporates a clustering method and ordered weighted averaging (OWA) operator. The proposed model can deal with (1) lack of persuasiveness in determining the universe of discourse and the linguistic length of intervals, and (2) the fact that only one attribute is usually considered in forecasting, not multiple attributes. Additionally, we use the owa operator to adjust error values and utilize previous periods to forecast the next period. Hence, not only does this project forecast one order but we can also deal with forecasting of high-order problems. Furthermore, we can, according to a different situation α , adjust forecasting values.

The rest of this project is organized as follows: section 2 introduces the related literature of the fuzzy time series model and Order Weighted Averaging (OWA) operator; in section 3, we demonstrate the proposed model and algorithm; section 4 evaluates the performance of the proposed model; and finally, conclusions are presented in section 5.

2. Related work

In this section, several related literatures, including fuzzy time series, order weighted averaging (OWA), and evaluation criteria, are briefly reviewed.

2.1 Fuzzy Time Series

Song and Chissom [6] proposed a fuzzy time series model to deal with the problems involving human linguistic terms [8-10]. In the following research, they continued to discuss the difference between time-invariant and time-variant models [6, 11]. Besides the above researchers, Chen presented a method to forecast the enrollment at the University of Alabama based on fuzzy time series [3]. Over the past 14 years, many fuzzy time series models have been proposed by following Song and Chissom's definitions [6, 11, 12]. Among these models, Chen's model is a very conventional one due to easy calculations and good forecasting performance [3, 5].

2.2 Information Fusion Technique: Ordered Weighted Averaging (OWA)

Yager proposed an ordered weighted averaging (OWA) operator that had the ability to get optimal weights of the attributes based on the rank of these weighting vectors after an aggregation process [13]. An OWA operator of dimension n is a mapping $f: R^n \rightarrow R$, which has an associated weighting vector $W = [w_1, w_2, \dots, w_n]^T$ with the following properties:

$W_i \in [0,1]$ for $i \in I = \{1,2,\dots,n\}$ and $\sum_{i \in I} W_i = 1$, such that

$$f(a_1, a_2, \dots, a_n) = \sum_{i \in I} W_i b_i \quad (1)$$

where b_i is the i th largest element in the collection. Thus, it satisfies the relation $Min_i[a_i] \leq f(a_1, a_2, \dots, a_n) \leq Max_i[a_i]$.

Fuller and Majlender [14] transformed Yager's OWA equation to a polynomial equation by using Lagrange multipliers. According to their approach, the associated weighting vector can be obtained by (2) ~ (4)

$$\ln w_j = \frac{j-1}{n-1} \ln w_n + \frac{n-j}{n-1} \ln w_1 \Rightarrow w_j = \sqrt[n-j]{w_1^{n-j} w_n^{j-1}} \quad (2)$$

$$\text{and } w_1 [(n-1)\alpha + 1 - n w_1]^n = \quad (3)$$

$$[(n-1)\alpha]^{n-1} [((n-1)\alpha - n)w_1 + 1]$$

if $w_1 = w_2 = \dots = w_n = \frac{1}{n} \Rightarrow \text{disp}(W) = \ln n$ ($\alpha = 0.5$) then

$$w_n = \frac{((n-1)\alpha - n)w_1 + 1}{(n-1)\alpha + 1 - n w_1} \quad (4)$$

where W_i is weight vector, N is number of attributes, and α is the situation parameter.

2.3 Evaluation criteria

MSE (Mean Squared Error) is used to measure performance. MSE is defined as eq (5).

$$MSE = \frac{\sum_{t=1}^n (Actual(t) - Forecast(t))^2}{n} \quad (5)$$

where actual(t) is the actual observations on time t, forecast(t) is the forecasting value, and n is the number of periods for forecasts.

3. The Proposed model

In order to deal with multiple attribute prediction and improve forecasting accuracy, this project hence proposes a hybrid model: (1) a clustering method (fuzzy c-mean [15]) for the partitioning process and (2) the OWA operator for high order adjusting.

3.1 Information Fusion Technique

In this section, we utilized an algorithm in order to calculate the OWA weights [16]. The main idea is from equation (2) ~ (4)[16]. The main procedure of the program is simply presented as follows (as Fig. 1).

For example, we compute the OWA weight under $n=3$ and situation $\alpha=0.5 \sim 1.0$ in Table 1

3.2 Algorithm of the proposed method

In this section, we utilize the yearly data on enrollments at the University of Alabama to present the proposed algorithm.

Step 1: Select attributes

According to research problem, select attributes by domain experts. From Table 2, the yearly enrollments of the University of Alabama only have one attribute; hence, we use enrollments as the main attribute.

Step 2: Computing the difference of each period

In this step, we calculate the difference of each period.

Step 3: Cluster multi-attribute time series S(t)

Supposed a time series S(t) with n observations of m attributes, an appropriate fuzzy clustering procedure, is selected to cluster time series S(t) into c ($2 \leq c \leq n$) clusters in this step. In this study, FCM is chosen because it is one of the fuzzy clustering methods.

According to (Miller, 1956), seven (linguistic value) is utilized as the number of clusters for demonstration to correspond with the limitation of human cognition in shortened memory. Hence, we use the difference from each period into FCM by equation (3).

Step 4: Rank each cluster and fuzzify the time series S(t) as fuzzy time series F(t)

In Table 2, we ranked each cluster center value and got the linguistic value.

Step 5: Fuzzify the historical data

Firstly, define the fuzzy set $L_1 L_2 \dots L_7$, on the universe of discourse.

Secondly, find out the degree of each observation value belonging to each L_i ($i=1, \dots, 7$). If the maximum membership of the observation value is under L_7 , then the fuzzified observation value is labeled as L_7 . Lastly, convert each stock price in the training dataset to corresponding linguistic values, L_7 . From Table 2, the maximum membership of the difference from 1971 and 1972 is 0.999773 in the fifth cluster.

Step 6: Establish fuzzy logic relationships and the fuzzy logic relationships groups

Construct a fuzzy logical relationship between two consecutive linguistic values, such as $A_i \rightarrow A_j$, where A_i is called the LHS (left-hand side) and A_j is the RHS (right-hand side) of the FLR. Then, we establish fuzzy logical relationship groups. For instance, A_i includes the fuzzy logical relationship $A_i \rightarrow A_j$, $A_i \rightarrow A_k$, and $A_i \rightarrow A_l$; hence, the fuzzy logical relationship group is $A_i \rightarrow A_j, A_k, A_l$.

Step 7: Defuzzify and compute forecast value

Suppose $F(t) = L_i$; the forecasting of $F(t+1)$ is conducted using the following rules.

Rule 1: According to the Naïve Forecasting Principle, if the fuzzy relationship group of L_i is empty, such as $L_i \rightarrow \text{empty}$, the forecasting of $F(t+1)$ is L_i .

Rule 2: If the fuzzy relationship group of L_i is one-to-one, such as $L_i \rightarrow L_x$, then the forecasting of $F(t+1)$ is L_x .

Rule 3: If the fuzzy relationship group of L_i is one-to-many, such as $L_i \rightarrow L_2, \dots, L_k$, then the forecasting of $F(t+1)$ supposes that each linguistic value is of equal weight. Therefore, the arithmetic mean is used as the forecasted value. The equation is as follows:

$$\frac{\text{Def}(L_1) + \text{Def}(L_2) + \dots + \text{Def}(L_k)}{k} \quad (6)$$

where $\text{Def}()$ is denoted as the defuzzified value of that linguistic value and k is the number of linguistic values.

Step 8: Calculate adjusted forecasting values based on the OWA operator

From 3.1, the attributes ordering and number are known. For computing the aggregated values, we multiply the values of the attributes ordering by corresponding to the weights of OWA. Hence, the adjusted forecasting value is defined:

$$\text{The adjusted forecasting value} = \text{actual_value} + \sum_{i=1}^i W_i x_{ik} \quad (7)$$

where W_i is the OWA operator of the i th input variable, and x_{ik} is the i th difference of forecasting. n is the number of rows in the database.

4. Verifications and comparisons

In this section, to illustrate how the proposed model performs fuzzy time series forecasting, practically collected TAIEX (Taiwan Stock Exchange Capitalization Weighted Stock Index) futures and TAIEX data are employed as experiment datasets. Finally, we use MSE as a performance indicator to compare the proposed model with Hang et al.'s [1] and Cheng et al.'s [2] models.

Forecasting TAIEX Futures

In this case, we practically collected TAIEX futures (2004/1/1~2004/12/15) as experiment datasets. From this dataset have 208 records, which are utilized 248 records (2004/1/1~2004/11/19) as training datasets; others are testing datasets (18 records, 2004/11/22~2004/12/15). To examine the improvement in performance, three fuzzy time series models, Hang et al.'s [1], and Cheng et al.'s [2]] models are employed as comparison models. The comparison result is shown in Table 3 and Table 4. We utilize an order from 3 to 7, and each order has an α value (0.5~1.0). Hence, we compute the average MSE (α value:0.5~1.0) as the performance of every order. Clearly, the performance of the proposed model surpasses the listing models.

Forecasting TAIEX

In this case, we practically collected TAIEX (2000/1/4~2001/12/31) data as experiment datasets. We utilized 10 months as training datasets, and 2 months are testing datasets. The comparison result is shown in Table 5 ~ Table 8. Clearly, the performance of the proposed model surpasses the listing models.

5. Conclusions

In this project, a high-order multiple attribute fuzzy time series method is proposed, which employs a fuzzy c-mean clustering method and OWA operator. From the experiment results, the performance of the proposed model surpasses the listing models. In order to enhance the forecasting accuracy, this project proposed an error adjust method to improve forecasting accuracy, and further utilized the OWA operator to decrease the data dimension. Moreover, the proposed model can overcome (1) a lack of persuasiveness in determining the universe of discourse and the linguistic length of intervals, and (2) that fact that only one attribute is usually considered in forecasting, not multiple attributes. Besides, we utilized a different situation α to adjust high order forecasting values. Finally, from the compared result, we can see that the proposed model outperforms the listing models. Based on the verification results, we conclude that the research goal has been reached.

6. 計畫成果自評

本研究內容與計畫內容大致相符，其目的均已達成。

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For different α and n , we can get different OWA weights. /* n is the number of attributes; α is the situation parameter */

OWA(n, α)

If $\alpha < 0.5$

Then $\alpha = 1 - \alpha$

If $\alpha > 0.5$

Then $w_i [(n-1)\alpha + 1 - n w_1]^n = [(n-1)\alpha]^{n-1} [((n-1)\alpha - n) w_1 + 1]$ //Calculate W_i

$w_n = [((n-1)\alpha - n) w_1 + 1] / [(n-1)\alpha + 1 - n w_1]$ //Calculate W_n

For $i = 2$ to $n-1$ do

$w_i = \sqrt[n-i]{w_1^{n-i} w_n^{i-1}}$ //Calculate W_i

Figure 1. Algorithm of OWA [16]

Table 1. The $w_1^* \sim w_3^*$ values for different situation values α

	$\alpha=0.5$	$\alpha=0.6$	$\alpha=0.7$	$\alpha=0.8$	$\alpha=0.9$	$\alpha=1$
w_1^*	0.333333	0.438355	0.553955	0.681854	0.826294	1
w_2^*	0.333333	0.323242	0.291992	0.23584	0.146973	0
w_3^*	0.333333	0.238392	0.153999	0.081892	0.026306	0

Table 2 Results of forecasting difference

Year	Enrollments	difference	Linguistic value	Forecasted Year	Enrollments	difference	Linguistic value	Forecasted
1971	13055		
1972	13563	508	5	297	199019328	358	4	387
1973	13867	304	4	387	199119337	9	3	230
1974	14696	829	6	612	199218876	-461	2	-465

Table 3 Results and comparisons for TAIEX forecasting (training)

Huang's [1]	Cheng et al.'s [2]	The proposed model				
		order				
		3	4	5	6	7
21950	20431	11768	11712	11689	11706	11629

Table 4 Results and comparisons for TAIEX forecasting (testing)

Huang's [1]	Cheng et al.'s [2]	The proposed model				
		order				
		3	4	5	6	7
32252	8298	3683	2755	2331	2309	2377

Table 5 Results and comparisons for TAIEX forecasting (training) in 2000

Huang's [1]	Cheng et al.'s [2]	The proposed model				
		order				
		3	4	5	6	7
115426	87696	31664	31488	31074	31363	31377

Table 6 Results and comparisons for TAIEX forecasting (testing) in 2000

Huang's [1]	Cheng et al.'s [2]	The proposed model				
		order				
		3	4	5	6	7
129007	373176	19260	19431	20128	20236	20617

Table 7 Results and comparisons for TAIEX forecasting (training) in 2001

Huarng's [1]	Cheng et al.'s [2]	The proposed model				
		order				
32882	19312	3	4	5	6	7
		8368	8330	8276	8268	8263

Table 8 Results and comparisons for TAIEX forecasting (testing) in 2001

Huarng's [1]	Cheng et al.'s [2]	The proposed model				
		order				
66615	15815	3	4	5	6	7
		13047	13245	13355	13517	13505

國科會補助教師出席國際學術會議報告書

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論文名稱	(中文)資訊融合技術於模糊時間序列 (英文) Information Fusion Technique for Fuzzy Time Series Model		

一、參加會議經過

本會議的主題是有關機器學習與電腦計算，恰巧與本人的這次資訊融合技術與模糊時間序列的研究方向相關，發表地點於澳洲柏斯。與會中本人與多位學者相互交流各國文化，彼此了解各國的人文風情，會議中主辦單位安排了一連串正式發表與學術研討的議程，內容十分充實。本次會議期間是從澳洲當地時間七月十日到七月十二日，環顧整個會場，各國研究精英齊聚一堂，相互交頭接耳、聚精會神地討論起各式新型演算法的貢獻與緣由，讓本人受益良多，無形之中我也感染了這種氛圍，了解許多各項新式演算法的使用，而且在不知不覺當中，我竟然慢慢地放鬆了身心，自然地與與會人事交流。研討會結束之後，並經由墨爾本、雪梨轉機回台，結束這難忘的研討會之旅。

二、與會心得

此次研討會最大的收穫，就是多了些機會與他國研究精英們共同討論研究，以改善了解我研究方面的不足，另一方面也開拓了我國際化研究的視野。我個人認為，本次研討會對於我日後赴他國，不論是發表演說，或從事共同研究性質方面的工作等等，是個難得的經驗；此外，由於這次的研討會，使得本人能夠更進一步地與各國演算法領域的專家相互交流研究心得，並且也深切地瞭解到他們的努力，與其優異性和其研究之重要性。

三、建議

希望日後能繼續推動國內研究學者出席國際研討會發表，讓更多台灣本土的研究學者，都能夠有更多的機會與國際接軌，讓我們的研究無時差。

四、攜回資料名稱及內容

此次會議主要攜回有 IACSIT ICMLC 2009 論文集、與會證明書與照片，內容主要是此次與會

的所有論文，與參與的證明，附件為本次投稿之論文。



六、其他

最後必須感謝國科會與南華大學給予以本人此次出國的補助款，讓本人能夠順利成行這次的研討會。

Information Fusion Technique for Fuzzy Time Series Model

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Abstract. This paper proposes a high order multiple attribute fuzzy time series model, which incorporate a clustering method and order weighted averaging (OWA) operator. The proposed model can deal with (1) lacking persuasiveness in determining the universe of discourse and the linguistic length of intervals, and (2) only one attribute is usually considered in forecasting not multiple attributes. Furthermore, we can accord different situation α to adjust high order forecasting value. To verify the proposed model, we use the yearly data on enrollments at the University of Alabama and TAIFEX (Taiwan Futures Exchange) as experimental datasets. Finally, this paper compares forecasting performances of proposed methods with Hang et al.'s [1] and Cheng et al.'s [2] models, the results of empirical analysis conclude that the proposed model surpasses in accuracy the listing models.

Keywords: Fuzzy time series, Order Weighted Averaging (OWA), Fuzzy clustering, Fuzzy logic relationship (FLR)

1. Introduction

In recent years, time-series models have utilized the fuzzy theory to solve various forecasting problems, such as university enrolment forecasting [3], financial forecasting [1, 4] and temperature forecasting [5]. Many researchers have proposed different methods to predict stock price based on fuzzy time series. In 1993, Song and Chissom[6] proposed the first fuzzy time series method, where the definitions, the time-invariant model[4] and the time-variant model[1] of fuzzy time series were presented. The following research proposed simple calculation methods to get a higher forecasting accuracy [3]. Huang and Yu utilized two methods, distribution-based and average-based length, to set the linguistic intervals of the universe of discourse to improve forecasting accuracy [1]. From the literature above, there are drawbacks: (1) Many factors in stock markets, such as financial report, macro economical data, and the fluctuations of foreign stock markets, influence practically the decisions of stock investors, and, therefore, multiple attributes should be considered in forecasting model. (2) In stock market forecasting, it is not reasonable to partition the universe of the discourse for dataset because the distribution of stock price is not uniform.

Therefore, this paper proposes a multiple attribute fuzzy time series model, which incorporate a clustering method and order weighted averaging (OWA) operator. The proposed model can deal with (1) lacking persuasiveness in determining the universe of discourse and the linguistic length of intervals, and (2) only one attribute is usually considered in forecasting not multiple attributes. Furthermore, we can according to different situation α to adjust forecasting value.

The rest of this paper is organized as follows: section 2 introduces the related literature of fuzzy time-series model, Order Weighted Averaging (OWA) operator; in section 3, demonstrates the proposed model and algorithm; section 4 evaluates the performance of the proposed model; and, finally, conclusions are presented in section 5.

2. Related work

In this section, several related literatures including Fuzzy time-series, order weighted averaging (OWA), and Evaluation criteria are briefly reviewed.

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2.1 Fuzzy Time-series

Song and Chissom [6] proposed a Fuzzy Time Series model to deal with the problems involving human linguistic terms [8-10]. In the following research, they continued to discuss the difference between time-invariant and time-variant models [6, 11]. Besides the above researchers, Chen presented a method to forecast the enrollments of the University of Alabama based on fuzzy time series [3].

Over the past fourteen years, many fuzzy time-series models have been proposed by following Song and Chissom's definitions [6, 11, 12]. Among these models, Chen's model is very conventional one because of easy calculations and good forecasting performance [3, 5]. Therefore, Song and Chissom's definitions and Chen's algorithm are used for illustrations as follows:

Definition 1: fuzzy time-series

Let $Y(t) (t = \dots, 0, 1, 2, \dots)$, a subset of real numbers, be the universe of discourse by which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t) \dots$ then $F(t)$ is called a fuzzy time-series defined on $y(t)$.

Definition 2: fuzzy time-series relationships

Assuming that $F(t)$ is caused only by $F(t-1)$, then the relationship can be expressed as: $F(t) = F(t-1) * R(t, t-1)$, which is the fuzzy relationship between $F(t)$ and $F(t-1)$, where $*$ represents as an operator. To sum up, let $F(t-1) = A_i$ and $F(t) = A_j$. The fuzzy logical relationship between $F(t)$ and $F(t-1)$ can be denoted as $A_i \rightarrow A_j$ where A_i refers to the left-hand side and A_j refers to the right-hand side of the fuzzy logical relationship. Furthermore, these fuzzy logical relationships can be grouped to establish different fuzzy relationships.

2.2 Information Fusion Techniques: Order Weighted Averaging (OWA)

Yager proposed an order weighted averaging (OWA) operator which had the ability to get optimal weights of the attributes based on the rank of these weighting vectors after aggregation process [13]. An OWA operator of dimension n is a mapping $f : R^n \rightarrow R$, that has an associated weighting vector $W = [w_1, w_2, \dots, w_n]^T$ with the following properties: $W_i \in [0, 1]$ for $i \in I = \{1, 2, \dots, n\}$ and $\sum_{i \in I} W_i = 1$, Such that

$$f(a_1, a_2, \dots, a_n) = \sum_{i \in I} W_i b_i \quad (1)$$

where b_i is the i th largest element in the collection. Thus, it satisfies the relation $Min_i[a_i] \leq f(a_1, a_2, \dots, a_n) \leq Max_i[a_i]$.

Fuller and Majlender [14] transform Yager's OWA equation to a polynomial equation by using Lagrange multipliers. According to their approach, the associated weighting vector can be obtained by (2) ~ (4)

$$\ln w_j = \frac{j-1}{n-1} \ln w_n + \frac{n-j}{n-1} \ln w_1 \Rightarrow w_j = \sqrt[n-1]{w_1^{n-j} w_n^{j-1}} \quad (2)$$

$$\text{and} \quad w_1 [(n-1)\alpha + 1 - n w_1]^n = (3)$$

$$[(n-1)\alpha]^{n-1} [((n-1)\alpha - n)w_1 + 1]$$

if $w_1 = w_2 = \dots = w_n = \frac{1}{n} \Rightarrow \text{disp}(W) = \ln n$ ($\alpha = 0.5$) then

$$w_n = \frac{((n-1)\alpha - n)w_1 + 1}{(n-1)\alpha + 1 - n w_1} \quad (4)$$

where w_i is weight vector, N is number of attributes, and α is the situation parameter.

2.3 Evaluation criteria

MSE (Mean Squared Error) is used to measure performance. MSE is defined as eq(5).

$$MSE = \frac{\sum_{i=1}^n (Actual(t) - Forecast(t))^2}{n} \quad (5)$$

where actual(t) is the actual observations on time t; forecast(t) is the forecasting value; and n is the number of periods for forecasts.

3. The Proposed model

In order to deal with multiple attribute prediction and improve forecasting accuracy, hence, this paper proposes a hybrid model: (1) clustering method (fuzzy c-mean [15]) for partitioning process, and (2) OWA operator for high order adjusting.

3.1 OWA weight

In this section, we utilized an algorithm in order to calculate the OWA weights [16]. The main idea is from equation (2) ~ (4)[16]. The main procedure of the program is simply presented as following (as fig. 1):

For different α and n , we can get different OWA weight. /* n is the number of attributes; α is the situation

parameter */

OWA(n, α)

If $\alpha < 0.5$

Then $\alpha = 1 - \alpha$

If $\alpha > 0.5$

Then $w_1[(n-1)\alpha + 1 - n w_1]^n = [(n-1)\alpha]^{n-1} [((n-1)\alpha - n)w_1 + 1]$ //Calculate W_1

$w_n = [((n-1)\alpha - n)w_1 + 1] / [(n-1)\alpha + 1 - n w_1]$ //Calculate W_n

For $i = 2$ to $n-1$ do

$w_i = \sqrt[n-i]{w_1^{n-i} w_n^{i-1}}$ //Calculate W_i

Figure 1. Algorithm of OWA [16]

For example, we compute OWA weight under $n=3$ and situation $\alpha=0.5\sim 1.0$ in table 1

Table 1. The $w_1^* \sim w_3^*$ values for different situation values α

	$\alpha=0.5$	$\alpha=0.6$	$\alpha=0.7$	$\alpha=0.8$	$\alpha=0.9$	$\alpha=1$
w_1^*	0.333333	0.438355	0.553955	0.681854	0.826294	1
w_2^*	0.333333	0.323242	0.291992	0.23584	0.146973	0
w_3^*	0.333333	0.238392	0.153999	0.081892	0.026306	0

3.2 Algorithm of the proposed method

In this section, we utilize the yearly data on enrollments at the University of Alabama to present the proposed algorithm.

Step 1: Select attributes

According to research problem, select attributes by domain experts. From table 2, the yearly enrollments of the University of Alabama only have one attribute; hence, we use enrollments as main attribute.

Step 2: Computing difference of each period

In this step, we calculate difference of each period. From table 1, difference of 1971 and 1972 is 508(=13563-13055). Therefore, we can get difference of each period in table 2.

Step 3: Cluster multi-attributes time series S(t)

Supposed a time series S(t) with n observations of m attributes, an appropriate fuzzy clustering procedure is selected to cluster time series S(t) into c ($2 \leq c \leq n$) clusters in this step. In this study, FCM is chosen because it is one of the fuzzy clustering methods.

According to (Miller, 1956), seven (linguistic value) is utilized as the number of clusters for demonstration to correspond with the limitation of human cognition in shorten memory. Hence, we use difference of each period into FCM by equation (3).

Step 4: Rank each cluster and fuzzify the time series S(t) as fuzzy time series F(t)

In table 2, we ranked each of cluster center value and got linguistic value.

Step 5: Fuzzify the historical data

Firstly, define the fuzzy set, L_1, L_2, \dots, L_7 , on the universe of discourse.

Secondly, find out the degree of each observation value belonging to each L_i ($i=1, \dots, 7$). If the maximum membership of the observation value is under L_7 , then the fuzzified observation value is labeled as L_7 . Lastly, convert each stock price in training dataset to corresponding linguistic values, L_7 . From table 2, the maximum membership of the difference of 1971 and 1972 is 0.999773 in fifth cluster.

Step 6: Establish fuzzy logic relationships and the fuzzy logic relationships groups

Construct fuzzy logical relationship between two consecutive linguistic values such as $A_i \rightarrow A_j$, where A_i is called the LHS (left-hand side) and A_j the RHS (right-hand side) of the FLR. Then, we establish fuzzy logical relationship groups. For instance, A_i include fuzzy logical relationship $A_i \rightarrow A_j, A_i \rightarrow A_k$, and $A_i \rightarrow A_l$, hence, the fuzzy logical relationship group is $A_i \rightarrow A_j, A_k, A_l$.

Step 7: Defuzzify and compute forecast value

Suppose $F(t) = L_i$, the forecasting of $F(t+1)$ is conducted using the following rules.

Rule 1: According to the Naïve Forecasting Principle, if the fuzzy relationship group of L_i is empty, such as $L_i \rightarrow \text{empty}$, the forecasting of $F(t+1)$ is L_i .

Rule 2: If the fuzzy relationship group of L_i is one-to-one, such as $L_i \rightarrow L_x$, then the forecasting of $F(t+1)$ is L_x .

Rule 3: If the fuzzy relationship group of L_i is one-to-many, such as $L_i \rightarrow L_2, \dots, L_k$, then the forecasting of $F(t+1)$ is supposed that each linguistic values is of the equal weight. Therefore, arithmetic mean is used as the forecasted value. The equation is as following:

$$\frac{\text{Def}(L_1) + \text{Def}(L_2) + \dots + \text{Def}(L_k)}{k} \quad (6)$$

where $\text{Def}()$ is denoted as the defuzzified value of that linguistic values and k is number of linguistic values.

From table 2, if we want to forecast the enrollments in 1973, fuzzy relation group $L_4 \rightarrow L_3, L_4, L_6$ is inferred. The forecasted value = (center of L_3 + center of L_4 + center of L_6)/3 = (297+24+840)/3 = 387

Table 2 Results of forecasting difference(part)

Year	Enrollments difference	Linguistic value	Forecasted	Year	Enrollments difference	Linguistic value	Forecasted		
1971	13055				
1972	13563	508	5	297	1990	19328	358	4	387
1973	13867	304	4	387	1991	19337	9	3	230
1974	14696	829	6	612	1992	18876	-461	2	-465

Step 8: Calculate adjusted forecasting values based on OWA operator

From 3.1, the attributes ordering and number are known. For computing the aggregated values, we multiply the values of attributes ordering by corresponding to the weights of OWA. For example, we calculate forecasting value in 1975 under $\alpha=0.5$ and $n=3$. Forecasting_value₁₉₇₅ = 15460 + 612 * 0.333333 + 387 * 0.333333 + 297 * 0.333333 = 15892

4. Verifications and comparisons

In this section, to illustrate how the proposed model performs fuzzy time series forecasting, practically collected TAIEX (Taiwan Stock Exchange Capitalization Weighted Stock Index) futures are employed as experiment datasets. Finally, we use MSE as performance indicator to compare proposed model with Hang et al.'s [1], and Cheng et al.'s [2] models.

Forecasting TAIEX Futures

In this case, we practically collect TAIEX futures (2004/1/1~2004/12/15) as experiment datasets. From this datasets have 208 records, which are utilized 248 records (2004/1/1~2004/11/19) as training datasets, others are testing datasets (18 records, 2004/11/22~2004/12/15). To examine the improvement in performance, three fuzzy time-series models, Hang et al.'s [1], and Cheng et al.'s [2] models, are employed as comparison models. The comparison result is shown in Table 3, MSE of the proposed model are 496.87. Clearly, the performance of the proposed model surpasses the listing models.

Table 3 Results and comparisons for TAIEX forecasting

		Type 2 method [1]	Cheng et al.'s[2]	The proposed model
MSE	Training data	21950.16	20431.44	488.81
	Testing data	32252.22	8297.627	496.87

5. Conclusions

In this paper, a high order multiple attribute fuzzy time series method is proposed which employs fuzzy c-mean clustering method and OWA operator. The proposed model can overcome (1) lacking persuasiveness in determining the universe of discourse and the linguistic length of intervals, and (2) only one attribute is usually considered in forecasting not multiple attributes. Besides, we utilize different situation α to adjust high order forecasting values. Finally, from the compare result, we can see that the proposed model outperforms the listing models. Based on the verification results, we conclude that the research goal has been reached.

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