# 行政院國家科學委員會專題研究計畫 成果報告

## 趨勢變異為基礎之模糊時間序列方法
## 研究成果報告(精簡版)

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計 畫 主 持 人：王佳文

計畫參與人員：博士班研究生-兼任助理：莊東燐
碩士班研究生-兼任助理：黃意純


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行政院國家科學委員會補助專題研究計畫 ☑ 成果報告 □期中進度報告

# 趨勢變異為基礎之模糊時間序列方法

計畫類別：☑ 個別型計畫 □ 整合型計畫

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共同主持人：

計畫參與人員：莊東燐、黃意純


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執行單位：南華大學電子商務學系


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## 中文摘要

過去傳統的時間序列方法雖然能夠對未來的事情做預測，但是卻無法處理語意型態的歷史資料。模糊時間序列方法是用以解決傳統方法之不足的方式之一，1993 年Song和Chissom提出了模糊時間序列方法用以解決傳統方法之不足。截至目前為止，已經有許多學者提出各式處理不確定性、不明確型態資料的模糊時間序列方法。本研究提出以趨勢變異為基礎的模糊時間序列，其包含了四個優點：(1) 依模糊分群來決定論域區間長度；(2)以漲跌幅趨勢來預測資料；(3)使用權重規則；(4)利用追蹤信號來比較其預測準確性。在實驗比較方面，本研究採用阿拉巴馬入學人口數與台灣指數期貨(TX)來驗證提出的方法，預期能夠比現存的方法更準確。

關鍵詞：模糊時間序列、台灣指數期貨(TX)、漲跌幅趨勢

## Abstract

Traditional time series methods fail to forecast the problems with linguistic historical data. An alternative forecasting method such as fuzzy time series is needed to deal with these kinds of problems. Song and Chissom (1993) proposed the fuzzy time series to solve the problems of the traditional time series methods. So far, many previous studies have proposed variant fuzzy time series models to deal with uncertain and vague data. This study proposes a fuzzy time series method based on trend variations. It has three advantages: (1) determining universe of discourse and the length of intervals based on fuzzy clustering; (2) forecasting data based on trend variations; (3) using the weighted rule; (4) utilizing the tracking signal to compare the forecasting accuracy. In experiments and comparisons, the enrollments at the University of Alabama and the Taiwan stock exchange capitalization weighted stock Index futures (TX) are adopted to illustrate and verify the proposed method, respectively. This study utilizes the tracking signal to compares the forecasting accuracy of proposed model with other methods, and the comparison results show that the proposed method has better performance than other methods.

***Keywords***: fuzzy time series, Taiwan stock exchange capitalization weighted stock Index futures (TX), trend variations

## 壹、前言

時間序列仰賴大量歷史資料的支持，藉由歷史資料推斷進而預測未來。時間序列一般來說有四個方向，其包括趨勢、週期、季節性和隨機變異(Wisner et al. 2006)。歷史性的資料過去都由傳統的時間序列處理，其傳統方法有些缺點： (1) 不能預測有關語意歷史資料的問題。(2) 傳統時間序列需要大量歷史資料，且資料必須是常態分配才能得到較好的效能。(3)傳統時間序列仰賴線性關係的支持 (Wisner et al. 2006)，但現實資料中，許多都存在著非線性，因此漸漸的許多研究利用模糊時間序列的方法來解決傳統時間序列的問題(Song & Chissom 1996；Chen 1996；Huarng 2001；Yu 2004)。

人們常用語意來表達日常觀察現象，為了符合人的思考邏輯，近年來很多模糊時間序列方法被提出，例如 Song 和 Chissom(1994)建構出的模糊關係矩陣，利用最大-最小運算子(max-min operator)計算出預測值；Chen(1996)利用均等區間長度來切割論域，利用簡單的計算來產生預測法則；Huarng(2001)延伸 Chen 的方法，且利用探索式的簡單計算，算出預測值；Yu(2004)提出加權方式來解決週期性的模糊關係等等。但在過去學者們所提出的時間序列方法都有共同的問題，除了對於宇集的決定與區間長度缺乏可信度之外，並無法處理多屬性模糊時間序列的預測，並且大多學者並無考量到趨勢漲跌的變化，也在結果方面並未考量到預測偏差的影響。

論域的分割（partition）與模糊叢集有許多相似之處，本研究應用叢集式模糊時間序列方法(Cheng et al. 2008)，並且加入漲跌幅的變化。本研究將藉由模糊分群來建立趨勢漲跌幅的變化的語言標籤，並用語言標籤建立預測的規則來加以訓練，使趨勢預測結果更穩定，最後利用追蹤信號(tracking signal)來做為評估準則，此訊號主要是在代表預測結果是否是偏態，預測方法是否會有偏高或邊低的結果產生，使用這樣的準則能讓結果更具合理性，因為預測的最終目的，是希望擁有準確與不偏的預測。最後在實驗比較方面，本研究利用阿拉巴馬大學入學人口數與台灣指數期貨數據進行驗證比較，並預期結果都能有效的預測且能夠比現存的方法更好。

# 貳、文獻探討

與本研究相關的研究共可分成幾方面來探討，分別是（1）模糊相關理論；（2）時間序列；（3）模糊分群，其分述如下。

## 2.1 模糊相關理論

模糊集合論是 1965 年由美國加州柏克萊大學扎德所提出來（Zadeh 1965），他強調人類的思維、推理以及對週遭事物的認知，其概念都是相當模糊的，他同時也認為許多傳統非常精確的數量方法，已經不能完全解決以人為中心的問題以及較為複雜的問題，為了面對這些現實環境中之不確定（uncertainty）與模糊性（fuzziness）資料，必須以模糊集理論的觀念來對應。

## 2.3 模糊時間序列

Zadeh 提出的模糊理論解決真實世界中普遍存在似是而非的模糊現象，當人面對定義不清楚或容易因個人認知不同而導致意義有所差異，可透過模糊數學方法，進行適當的定量分析處理問題，Song, Q. 和 Chissom, B. S. (1993a) 基於這樣的觀念針對語意資料提出了模糊時間序列的預測模型。他用不同的三角模糊數方法模糊明確值下列描述相關模糊時間數列的定義：

【定義 1】模糊時間序列

令 $Y(t)(t=...,0,1,2,...)$ 為宇集且為 R 的部份集合，且 $u_i(t)(i=1,2,...)$ 為定義在宇集 $Y(t)$ 中之模糊集合。如果 $F(t)$ 為 $Y(t)(t=...,0,1,2,...)$ 的集合，則 $F(t)$ 為 $Y(t)(t=...,0,1,2,...)$ 的模糊時間序列。

【定義 2】模糊時間序列關係

令 $F(t)$ 為一時間序列且存在模糊關係 $R(t-1,t)$，使得 $F(t)=F(t-1)\times R(t-1,t)$，其中”×”為運算子，且 $F(t-1)$ 導致 $F(t)$，表示成 $F(t-1)\rightarrow F(t)$。

【定義 3】時間變動性與非時間變動性模糊時間序列

假設 $F(t)$ 由 $F(t-1)$ 所導致，使得 $F(t)=F(t-1)\times R(t-1,t)$，對任一 t，

假使模糊關係 $R(t-1,t)$ 與 t 獨立，則稱 $F(t)$ 為非時間變動性模糊時間序列，反之則稱為時間變動性模糊時間序列。

Song, Q. 和 Chissom, B. S. 於 1994 年的研究中，也探討非時間變動性與時間變動性模型的差異，提出三層倒傳遞神經網路架構轉換輸出並比較三種不同的解模糊方法，他們的結果顯示類神經網路解模糊模型有較小的平均預測誤差，但其所提出的方法運用 Max-Min 運算子，需要非常大量且複雜的計算。另一方面，Chen 於 1996，利用簡單的算術運算就可以得到很好的預測結果，其也使用 Alabama 大學新生入學的資料評估所提出的方法，結果顯示他的方法不但能有效預測入學人數，而且不受干擾資料的影響。Huarng(2001)指出在時間序列中，區間長度會影響預測結果。其提出分配基準(distribution-based)和平均基準(averge-based)長度來接近這議題。其應用的區間長度在 Chen 模式上(1996)。這結果顯示了，分配基準和平均基準長度會改進預測結果。雖然 Huarng(2001)提出兩個方法來決定有效的區間長度，他將論域切了太多區間了，而有太多的語意值需要人類來辨識。根據 Miller 的研究(1956)語意值切割區間數目多寡，會影響人類和預測正確性的衡量。

## 2.3 預測準確性評估

Y 任何預測的最終目的，就是希望擁有準確與不偏的預測，因此有許多預測誤差被提出，本小節主要針對平均絕對誤差(MAD)、平均絕對百分比誤差(MAPE)、預測誤差移動總和(RSFE)和追蹤信號(Tracking Signal，TS)，公式說明如下(Wisner et al. 2005)：

(1) 平均絕對誤差(MAD)

$$\text{MAD}=\frac{\sum|e_t|}{n} \tag{1}$$

其 $e_t$ =第 t 期預測誤差，$e_t$ =第 t 期真正值-第 t 期預測值，n 為評估期數。

(2) 平均絕對百分比誤差(MAPE)

$$\text{MAPE}=\frac{1}{n}\sum\left|\frac{e_t}{A(t)}\right|(100) \tag{2}$$

其 $e_t$ =第 t 期預測誤差，n 為評估期數，$A(t)$ =第 t 期真正值。

(3) 預測誤差移動總和(RSFE)

$$\text{RSFE}=\sum e_t \tag{3}$$

其 $e_t$ =第 t 期預測誤差。

RSFE 主要是在衡量預測方法持續高估或低估，若不會偏向高估或低估，則偏態會在 0 附近跳動。

(4) 追蹤信號(Tracking Signal，TS)

$$\text{追蹤信號}=\frac{RSFE}{MAD} \tag{4}$$

追蹤信號是偏態和 MAD 的比值，此訊號主要是在代表預測是否是偏態的。

## 2.4 模糊分群

叢集（clustering）的方法經常用來對資料做分析以瞭解資料的結構，其方法是定義 C 個特徵向量 $v_j(j=1,2,....,C)\in R^C$ ，利用相似性測量與相關的目標函數將樣本 $x_k$ 分類至第 j 個群集中。Bezdek 首先提出模糊理論應用於群集分析的模糊 K 平均法（Fuzzy C Means，FCM），FCM 藉由群組間最小誤差目標函數將資料集做模糊分割，目標函式如下表示：

$$J_m(X,U,V)=\sum_{i=1}^{c}\sum_{k=1}^{n}u_{ik}^m d^2(x_k;v_i) \tag{5}$$

其中 $u_{ik}$ 與 $v_i$ 分別表示歸屬程度與中心座標，m 為模糊度指標，n 為特徵向量 $x_k$ 的個數，c 為群集數目，$d(x_k;v_i)$ 為資料與中心的相似程度。目標函式 $J_m$ 有下列的限

制：

$$(1) 0 \le u_{ik} \le 1, \forall i, k,$$

$$(2) 0 < \sum_{k=1}^{n} u_{ik} \le n, \forall i, \quad (6)$$

$$(3) \sum_{i=1}^{c} u_{ik} = 1, \forall k,$$

根據下列的表示更新中心$v_i$所組成的$v_{il}$與歸屬程度$u_{ik}$：

$$v_{il} = \frac{\sum_{k=1}^{n} u_{ik}^{m} x_{kl}}{\sum_{i=1}^{n} u_{ik}^{m}} \text{ 與 } u_{ik} = \frac{1}{\sum_{j=1}^{c} \left(\frac{d(x_k; v_i)}{d(x_k; v_j)}\right)^{2/m-1}} \quad (7)$$

## 參、趨勢變異為基礎之模糊時間序列方法

本章節共可分成二個子章節，分別是 3.1 研究架構，3.2 演算步驟，其分述如下。

### 3.1 研究架構

本研究將藉由趨勢漲跌與模糊分群來建立語言標籤，並進而產生預測規則，使其結果更穩定減少預測偏差，最後利用評估指標追蹤信號來使預測結果更具合理化，在實證方面，本研究利用阿拉巴馬大學入學人口數與台灣指數期貨(TX)，做為實證。圖 1 為本研究架構。

### 3.2 演算步驟

**步驟一**:計算漲跌趨勢$D_t = A(t-1) - A(t)$，$A(t)$為$t$期真實值，$A(t-1)$為$t-1$期真實值

**步驟二**:利用 FCM 將$D_t$分群到個$k$叢集中$C_i\ (i = 1,2,3,\ldots,k)$。

**步驟三**：各叢集$C_i$依叢集中心$c_i\ (i = 1,2,3,\ldots,k)$大到小排序並給予語言標籤$L_r\ (r = 1,2,3\ldots,k)$。

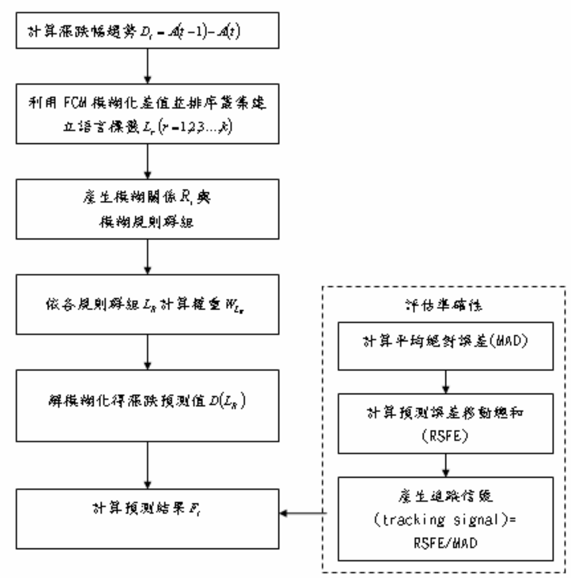


圖 1、一新模糊時間序列模式

**步驟四**:利用語言標籤建立模糊關係$L_R \to L_r\ (R = 1,2,3\ldots,k; r = 1,2,3\ldots,k)$與模糊關係規則群組$L_R$作預測，例如$L_1 \to L_2$，$L_1 \to L_2$，$L_1 \to L_3$，$L_1 \to L_4$則可簡化成$L_1 \to L_{2(2)}, L_3, L_4$，$L_{2(2)}$代表$L_2$在此群組出現過 2 次，其於$L_3$，$L_4$各出現 1 次。

**步驟五**:依各規則群組$L_R$計算權重$W_{L_R}$

$$W_{L_R} = \left[w_{L_1}, w_{L_2}, \ldots, w_{L_k}\right] = \left[\frac{\|L_R \to L_1\|}{\sum_{r=1}^{k}\{\|L_R \to L_r\|\}}, \frac{\|L_R \to L_2\|}{\sum_{r=1}^{k}\{\|L_R \to L_r\|\}}, \cdots, \frac{\|L_R \to L_r\|}{\sum_{r=1}^{k}\{\|L_R \to L_r\|\}}\right],$$

$$R = 1,2,3\ldots,k, \|\bullet\| \text{ 為基數} \quad (8)$$

**步驟六**:進行解模糊化得到預測漲跌預測值

$$D(L_R) = \sum_{i=1}^{k} c_i \times w_{L_i}, k \text{ 為叢集個數} \quad (9)$$

**步驟六**：依前期資料加上漲跌預測值得到最終預測結果

$F_t = A(t-1) - D(L_R)$，$F_t$ 為第 $t$ 期預測值 (10)

**步驟七**：評估準確性並與其他方法比較

## 肆 實驗比較

本章節共可分成二部份，分別是 4.1 阿拉巴馬大學入學人口數與 4.2 節台灣指數期貨，分別呈現如下。

### 4.1 阿拉巴馬大學入學人口數

以阿拉巴馬大學 1971~1992 入學人口數為例，表 1 為入學人口數真實值，經由步驟 4 可得表 2 規則庫，由實驗結果表 3 可得知本研究 MSE 為 193373，相較於過去之研究 MSE 為最小且追蹤信號(TS)為-0.08 為接近 0，因此可知，其結果優於所列之過去研究。最後本研究利用每日台灣指數期貨為實驗對象，結果呈現於 4.2 小節。

### 4.2 台灣指數期貨數據

在台灣指數期貨驗證中，本研究與過去有相同例子之研究進行比較(Cheng et al. 2008; Huarng & Yu 2005)，本研究以每日台灣指數期貨為實驗對象，其以 2004/1/28 ~ 2004/11/19 為訓練資料共 208 筆，2004/11/22~2004/12/15 為測試資料，共 18 筆。並採用收盤價為主要屬性而最高價與最低價為次要屬性來進行預測。其結果呈現於表 4 至表 6。表 4 為建立之規則，表 5 為訓練資料結果，表 6 為測試資料結果，以測試資料為例，可發現與過去有相同例子之研究進行比較時，其 MSE 值為最小，且 TS 值相對較接近 0，因此可知，其結果優於所列之過去研究。

## 伍、結論

本研究提出以趨勢變異為基礎的模糊時間序列，其包含了四個優點：(1)依模糊分群來決定論域區間長度；(2)以漲跌幅趨勢來預測資料；(3)加入權重規則;(4)利用追蹤信號來比較其預測準確性。在實驗比較方面，本研究採用阿拉巴馬入學人口數與台灣指數期貨來驗證提出的方法，由結果得知其能夠比本研究所列之研究更準確。

## 陸、計畫成果自評

本研究內容與計畫內容大致相符，其目的均已達成。

表 1 阿拉巴馬大學 1971~1992 入學人口數

| 年代 | 真實入學人數 | Linguistic | Forecasted |
|---|---|---|---|
| 1971 | 13055 | | |
| 1972 | 13563 | $L_5$ | |
| 1973 | 13867 | $L_4$ | 13859.51 |
| 1974 | 14696 | $L_6$ | 14253.93 |
| … | … | … | … |
| 1992 | 18876 | $L_2$ | 19567.09 |

表 2 規則庫

| 權重未經正規化之規則 |
|---|
| $L_1 \to L_3$ |
| $L_2 \to L_1, L_3$ |
| $L_3 \to L_{2(3)}, L_4, L_6$ |
| $L_4 \to L_3, L_4, L_{6(2)}$ |
| $L_5 \to L_4$ |
| $L_6 \to L_{3(2)}, L_4, L_{6(2)}, L_7$ |
| $L_7 \to L_6$ |

表 3 實驗結果比較

| | MSE | MAD | RSEF | TS |
|---|---|---|---|---|
| Chen (1996) | 427113.85 | 517.55 | -1491.00 | -2.88 |
| Song and Chissom (1994) | 423026.65 | 516.35 | -1387.00 | -2.69 |
| Cheng et al. (2008) | 231429.74 | 412.46 | 263.81 | 0.64 |
| Proposed Model | 193373 | 378.68 | -30.93 | -0.08 |

表 4 規則庫

| 權重未經正規化之規則 |
|---|
| $L_1 \rightarrow L_1, L_3, L_4, L_7$ |
| $L_2 \rightarrow L_1, L_{2(5)}, L_{3(5)}, L_{4(6)}, L_{5(2)}, L_{6(2)}, L_7$ |
| $L_3 \rightarrow L_{2(4)}, L_{3(5)}, L_{4(12)}, L_{5(11)}, L_{6(6)}, L_{7(4)}$ |
| $L_4 \rightarrow L_{2(6)}, L_{3(14)}, L_{4(7)}, L_{5(12)}, L_{6(5)}, L_{7(3)}$ |
| $L_5 \rightarrow L_{1(2)}, L_{2(2)}, L_{3(12)}, L_{4(10)}, L_{5(14)}, L_{6(7)}, L_{7(4)}$ |
| $L_6 \rightarrow L_{2(4)}, L_{3(1)}, L_{4(11)}, L_{5(6)}, L_{6(2)}, L_{7(1)}$ |
| $L_7 \rightarrow L_{2(1)}, L_{3(4)}, L_{4(1)}, L_{5(6)}, L_{6(2)}, L_{7(1)}$ |

表 5 2004/1/28 ~ 2004/11/19 訓練資料結果

| | MSE | MAD | RSEF | TS |
|---|---|---|---|---|
| Proposed | 11158.25 | 71.99 | 5.71 | 0.08 |

表 6 2004/11/22~2004/12~15 測試資料結果

| | MSE | MAD | RSEF | TS |
|---|---|---|---|---|
| Huarng &Yu (2005) | 32252.22 | 164.78 | 2288.00 | 13.89 |
| Cheng et al. (2008) | 8297.63 | 69.35 | 428.57 | 6.18 |
| Proposed Model | 5241.04 | 48.76 | -20.51 | -0.42 |

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# 出席國際學術會議心得報告

| | |
|---|---|
| 計畫編號 | NSC 96-2218-E-343-001 |
| 計畫名稱 | 趨勢變異為基礎之模糊時間序列方法 |
| 出國人員姓名<br>服務機關及職稱 | 王佳文，南華大學電子商務學系，助理教授 |
| 會議時間地點 | 時間 2007.08.19-22 地點香港 |
| 會議名稱 | International conference on Machine Learning and Cybernetics 2007 |
| 發表論文題目 | Information Fusion Techniques for Weighted Time Series Model |

一、 參加會議經過

本會議的主題是有關機器學習與控制，而本人參與的 session 是關於智慧型系統。於會中與多位學者相互交流各國文化，彼此了解各國的人文風情，會議中主辦單位安排了一連串正式發表與學術研討的議程，內容十分充實，本次會議期間是從香港當地時間 8 月 19 日至 8 月 22 日，由於本次會議所接受之研究會被收錄於 EI 研討會，所以參與的文章都有一定的水準。環顧整個會場，各國研究精英齊聚一堂，相互交頭接耳、聚精會神地討論起各式新型演算法的貢獻與緣由，讓本人受益良多，無形之中我也感染了這種氛圍。研討會結束之後，便搭機回台，結束這難忘的研討會之旅。

二、與會心得

此次研討會最大的收穫，就是多了些機會與他國研究精英們共同討論研究，以改善了解我研究方面的不足，另一方面也開拓了我國際化研究的視野。我個人認為，本次研討會對於我日後赴他國，不論是發表演說，或從事共同研究性質方面的工作等等，是個難得的經驗；此外，由於這次的研討會，使得本人能夠更進一步地與各國演算法領域的專家相互交流研究心得，並且也深切地瞭解到他們的努力，與其優異性和其研究之重要性。

# INFORMATION FUSION TECHNIQUE FOR WEIGHTED TIME SERIES MODEL

**JIA-WEN WANG[1], CHING-HSUE CHENG[2]**

[1]Departmetn of Electronic Commerce Management, Nanhua University, Taiwan
[2]Department of Information Management, National Yunlin University of Science & Technology, Taiwan
E-MAIL: jwwang@mail.nhu.edu.tw[1], chcheng@yuntech.edu.tw[2]

**Abstract:**

**In this paper, we propose an information fusion technique for weighted time Series Model, is called OWA-MA forecasting model. The OWA-MA forecasting model combines OWA operator and weighted moving average (WMA). The model deals with the dynamical weighting problem more rationally and flexibly according to the situational parameter α value from the user's viewpoint. For verifying proposed method, we use two datasets to illustrate our performance, the datasets are: (1) dataset 1: the yearly data on enrollments at the university of Alabama [5] and (2) dataset 2: the forecast demand table to evaluate the proposed model [6]. Furthermore, the tracking signal as evaluation criteria to compares the proposed model with other models. It is shown that our proposed method proves better than other methods for time series model.**

**Keywords: Information fusion technique; OWA operator, OWA-MA forecasting model, time series.**

## 1. Introduction

A sequence of n observed data, consisting of continuous values changing with time, is called a time series. Based on the idea that the time series carry within them the potential for predicting their future behaviors, the goal of time series prediction or forecasting is to find the continuations, $A_{n+1}$, of the observed n sequence $A_1$, $A_2$, $\cdots$, $A_3$. Time series contains multiple Periods, and how to get the relative weights in multi-criteria decision making (MCDM) problems is a very important issue [2, 4, 7, 8].

The ordered weighted averaging (OWA) issued by Yager [7] have been more popular recently. As history data is becoming largely available, we can consider different effects between the different Periods and to extract relevant information from the data. The OWA operators can fusion multi-Period values into aggregated values of single value. It can be applied to give different weight for every Period.

The weighted moving average forecasting model allows emphasis to be placed on more recent data to reflect changes in patterns. But weighted used also tend to be based on experience of the forecaster [4]. The main objective of the paper is to apply an information fusion technique for weighted Time Series Model, is called OWA-MA forecasting model. The OWA-MA forecasting model combines OWA operator and weighted moving average (MA) forecasting model. It not only considers the degree of different influence between different Periods but also focus on dynamical weighting problems. The decision makers (doctors) just need to change the weights of Periods dynamical, then the proposed method can revise the weight of each Periods based on aggregation situation and the system will provide forecasting results to the decision makers.

The rest of this paper is organized as follows. Section 2 is preliminaries are introduced. In section 3, the OWA-MA time series model is developed. In Section 4, we describe briefly how the processing of the OWA-MA time series model to implement. And this paper uses the yearly data on enrollments at the University of Alabama and the dataset of product demand to evaluate the proposed model. The conclusions and future research are discussed in Section 5.

## 2. Preliminaries

In this Section, we describe briefly about time series models, OWA Operator and forecast Accuracy.

### 2.1. Time Series Models

Time series forecasts are dependent of the availability of historical data. Forecasts are estimated by extrapolating the past data into the future [1, 6]. Time series data typical have four patterns: (1) trend variations, (2) Cyclical variations, (3) seasonal, and random variations [6]. Time series forecasting is one of the most widely used techniques. From the literatures, it indicates that the top tree quantitative forecasting techniques used are simple moving average, weighted moving average, and exponential smoothing. In this section, we introduce the basic definition as follows:

**[Definition 1]** Simple moving average (MA) forecasting model

$$F_{t+1} = \frac{\sum_{i=t-n+1}^{t} A_i}{n}, \tag{1}$$

where $F_{t+1}$ is forecast for Period t+1, $n =$ number of periods used to calculate moving average, and $A_i =$ actual demand in Period i.

**[Definition 2]** Weighted moving average (WMA) forecasting model

$$F_{t+1} = \sum_{i=t-n+1}^{t} w_i A_i, \tag{2}$$

where $F_{t+1}$ is forecast for Period t+1, $n =$ number of periods used to calculate moving average, $A_i =$ actual demand in Period I, and $w_i$ =weight assigned to Period I (with $\sum w_i = 1$).

**[Definition 3]** Exponential smoothing forecasting model

$$F_{t+1} = F_{t+1} + \beta(A_t - F_t), \tag{3}$$

where $F_{t+1}$ is forecast for Period t+1, $F_t$ is forecast for Period t, $A_i =$ actual demand in Period I, and $\beta$ =a smoothing constant $(0 \le \beta \le 1)$.

### 2.2. OWA Operator

The concept of OWA operators was first introduced by Yager in 1988 [8]. Many approaches have been proposed to calculate the weights based on OWA operators and apply this concept to many fields. In this section, we introduce the basic definition and some operations of OWA [2, 7, 8].

**[Definition 4]** An OWA operator of dimension n is a mapping F: $R^n \longrightarrow$ R, that has an associated weighting vector $W^* = \left[w_1^* w_2^* \ldots w_n^*\right]^T$ of having the properties

$$\sum_i w_i^* = 1, \forall w_i^* \in [0,1],\ i = 1, \ldots, n,$$

and such that $f(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j^* b_j$ (4)

where $b_j$ is the $j$th largest element of the collection of the aggregated objects $a_1, a_2, \ldots, a_n$.

Fuller and Majlender use the method of Lagrange multipliers to transfer equation (7) to a polynomial equation, which can determine the optimal weighting vector. By their method, the associated weighting vector is easily obtained by (8)-(9) [4].

$$\text{Orness}(W^*) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i^* \tag{5}$$

$$\text{Disp}(W^*) = -\sum_{i=1}^{n} w_i^* \ln w_i^* \tag{6}$$

Maximize $\sum_{i=1}^{n} w_i^* \ln w_i^*$,

Subject to $\alpha = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)\, w_i$ (7)

$$\ln w_j^* = \frac{j-1}{n-1}\ln w_n^* + \frac{n-j}{n-1}\ln w_1^*$$

$$\Rightarrow w_j^* = \sqrt[n-1]{w_1^{*n-j}\, w_n^{*j-1}} \quad (8)$$

and $w_1^*\left[(n-1)\alpha + 1 - nw_1^*\right]^n$

$$= \left[(n-1)\alpha\right]^{n-1}\left[((n-1)\alpha - n)w_1^* + 1\right] \quad (9)$$

If $w_1^* = w_2^* = \dots = w_n^* = \frac{1}{n} \Rightarrow$ disp($W^*$)=ln$n$

$$w_n^* = \frac{((n-1)\alpha - n)w_1^* + 1}{(n-1)\alpha + 1 - nw_1^*} \quad (10)$$

Hence, the optimal value of $w_1^*$ should satisfy equation (9). When $w_1^*$ is computed, we can determine $w_n^*$ from equation (10), and then the other weights are obtained from equation (8). In a special case, when $w_1^* = w_2^* = \dots = w_n^* = \frac{1}{n} \Rightarrow$ disp( $W^*$ ) = $\ln n$ which is the optimal solution for $\alpha = 0.5$.

The parameter $\alpha$ can be treated as a magnifying lens for the optimistic decision makers to determine the most important attribute based on the sparest information (*i.e.* optimistic and $\alpha$ =0 or 1) situation. On the other hand, when $\alpha$ =0.5 (moderate situation), this method can get the attributes' weights (equal weights of attributes) for the pessimistic decision makers based on maximal information (maximal entropy) [2].

### 2.3. Forecast Accuracy

The ultimate goal of any forecasting endeavor is to have an accurate and unbiased forecast [1, 5]. In this section, we introduce the formula for forecast error, defined as the difference between actual quantity and the forecast error, it is shown as follows:

1. Mean absolute deviation (MAD)

$$\text{MAD}=\frac{\sum|e_t|}{n} \quad (11)$$

Where $e_t$ = forecast error for Period t, $e_t = A_t - F_t$, $A_t$ =actual demand for Period $t$, and n=number of periods of evaluation.

2. Mean absolute percentage error (MAPE)

$$\text{MAPE}=\frac{1}{n}\sum\left|\frac{e_t}{A_t}\right|(100) \quad (12)$$

Where $e_t$ = forecast error for Period t, $e_t = A_t - F_t$, $A_t$ =actual demand for Period $t$, and n=number of periods of evaluation.

3. Running sum of forecast errors (RSFE)

$$\text{RSFE}=\sum e_t \quad (13)$$

Where $e_t$ = forecast error for Period t.

4. Tracking Signal

$$\text{Tracking Signal} = \frac{RSFE}{MAD} \quad (14)$$

The RSFE is an indicator of bias in the forecasts. The tracking signal is checked to determine if it is within the acceptable control limits [5]. The value is more closer zero is better.

## 3. The OWA-MA forecasting model s

The paper proposes information fusion technique for weighted time series model, it utilizes OWA based moving average time series model to solve time series problem. The study has three research objectives in the following:
(1)Apply OWA to get aggregated Periods t.
(2)Propose an OWA based moving average (OWA-MA) time series model to effectively solve the problem of time series.
(3)Verify the proposed approach by the yearly data on enrollments at the University of Alabama [5] and the dataset of product demand [6].

### 3.1. OWA operator algorithm

In this section, the main procedure of the OWA program is simply presented as following (as Figure 1.).

For different $\alpha$ and *n*, we can get different OWA weight.

OWA (n, $\alpha$)

/* n is the number of attributes; $\alpha$ is the situation parameter */

If $\alpha < 0.5$

Then $\alpha = 1 - \alpha$

If $\alpha > 0.5$

Then $w_1^*[(n-1)\alpha + 1 - nw_1^*]^n =$

$[(n-1)\alpha]^{n-1}\,[((n-1)\alpha - n)w_1^* + 1]$ //Calculate $w_1^*$

$w_n^* = [((n-1)\alpha - n)w_1^* + 1]/[(n-1)\alpha + 1 - nw_1^*]$

//Calculate $w_n^*$

Figure 1. Algorithm of the OWA operator

For example, if n=4 then we can get $w_1^* \sim w_4^*$, it is shown as Table 1. The distribution of the $w_1^* \sim w_4^*$ values for different situation parameter values is shown in Figure 2.

Table 1. The $w_1^* \sim w_4^*$ values for different situation parameter values $\alpha$

| | α=0.5 | α=0.6 | α=0.7 |
|---|---|---|---|
| $w_1^*$ | 0.25 | 0.416657 | 0.493805 |
| $w_2^*$ | 0.25 | 0.233398 | 0.237305 |
| $w_3^*$ | 0.25 | 0.130859 | 0.11377 |
| $w_4^*$ | 0.25 | 0.073547 | 0.054918 |
| | α=0.8 | α=0.9 | α=1.0 |
| $w_1^*$ | 0.59646 | 0.76409 | 1.00 |
| $w_2^*$ | 0.25195 | 0.18212 | 0.00 |
| $w_3^*$ | 0.10644 | 0.04345 | 0.00 |
| $w_4^*$ | 0.04501 | 0.01036 | 0.00 |

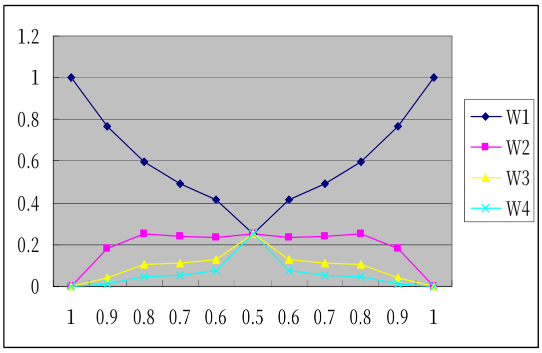


Figure 2. The $w_1^* \sim w_4^*$ values for different situation parameter values $\alpha$

### 3.2 The algorithm of OWA-MA forecasting model

In this section, the algorithm of OWA-MA is proposed as follows:

STEP 3.1: Given the period number *n* and situation parameter $\alpha$.

STEP 3.2: Rank the important degree of Period t. $(i.e., t_n > t_{n-1} > \cdots t_1)$

STEP 3.3: Get OWA weights $w_1^* \sim w_n^*$ by OWA algorithm.

STEP 3.4: Calculate the $F_{t+1}$ by the OWA-MA forecasting model.

$$F_{t+1} = \sum_{i=t-n+1}^{t} w_i^* A_i \,, \qquad (15),$$

where $F_{t+1}$ is forecast for Period t+1, $n =$ number of periods used to calculate moving average, $A_i =$ actual demand in Period I, and $w_i^*$=OWA weight assigned to Period i (with $\sum w_i^* = 1$).

STEP 3.5: Calculate the forecast accuracy by equations (11)~(14).

### 4. Verification and Comparison

For verification and comparison, the paper uses two databases: (1) the yearly data on enrollments at the University of Alabama and (2) the dataset of product demand to evaluate the proposed model.

#### 4.1 The yearly data on enrollments at the University of Alabama

This study adopts the yearly data on enrollments at the University of Alabama. The database contains a total of 22 periods as listed in Table 2. For example, calculate the forecast for Period 5 using a four-period OWA-MA.

(1) From step 3.1, we obtain the period number n=4 and situation parameter $\alpha = 0.8$.

(2) Secondly, we rank the important degree of Period t, then we can obtain $t_4 > t_3 > t_2 > t_1$.

(3) From OWA algorithm, we can get $w_1^* \sim w_4^*$, the results is shown as Table 1.

(4) From step 3.6, calculate the $F_5$ by the OWA-MA forecasting model.

If $\alpha = 0.8$ then $w_1^* \sim w_4^*$ =0.596466, 0.251953, 0.106445, 0.045018.

$$F_5 = (0.596466*14696 + 0.251953*13867 + 0.106445*13563 + 0.045018*13055)/4$$
$$=14290.93$$

(5) Calculate the forecast accuracy by equations (11)~(14). The results are shown as Table 4 and Table 5.

Finally, we compute tracking signal to evaluate the estimated accuracy. The tracking signal is more closer zero is better. In Table 4 and Table 5, we can see that the MAPE is smaller than MA and WMA, and the tracking signal of the proposed approach with OWA operator is closer zero, that is, the proposed model is more robust than MA and WMA.

Table 2. The yearly data on enrollments at the University of Alabama

| Year | Actual Enrollment | Year | Actual Enrollment |
|---|---|---|---|
| 1971 | 13055 | 1982 | 15433 |
| 1972 | 13563 | 1983 | 15497 |
| 1973 | 13867 | 1984 | 15145 |
| 1974 | 14696 | 1985 | 15163 |
| 1975 | 15460 | 1986 | 15984 |
| 1976 | 15311 | 1987 | 16859 |
| 1977 | 15603 | 1988 | 18150 |
| 1978 | 15861 | 1989 | 18970 |
| 1979 | 16807 | 1990 | 19328 |
| 1980 | 16919 | 1991 | 19337 |
| 1981 | 16388 | 1992 | 18876 |

Table 3. The results of the proposed model for dataset 1

| Year | Actual Enrollment | Forecast Enrollment | MAD | RSFE |
|---|---|---|---|---|
| 1971 | 13055 | | | |
| 1972 | 13563 | | | |
| 1973 | 13867 | | | |
| 1974 | 14696 | | | |
| 1975 | 15460 | 14290.93 | 1169.073 | 1169.073 |
| 1976 | 15311 | 15010.72 | 300.2755 | 300.2755 |
| 1977 | 15603 | 15216.27 | 386.7282 | 386.7282 |
| 1978 | 15861 | 15471.54 | 389.457 | 389.457 |
| 1979 | 16807 | 15717.53 | 1089.465 | 1089.465 |
| 1980 | 16919 | 16371.17 | 547.8299 | 547.8299 |
| 1981 | 16388 | 16716.93 | 328.9301 | -328.93 |
| 1982 | 15433 | 16540.74 | 1107.737 | -1107.74 |
| 1983 | 15497 | 15891.83 | 394.834 | -394.834 |
| 1984 | 15145 | 15637.91 | 492.9121 | -492.912 |
| 1985 | 15163 | 15318.52 | 155.5213 | -155.521 |
| 1986 | 15984 | 15204.39 | 779.6095 | 779.6095 |
| 1987 | 16859 | 15664.04 | 1194.963 | 1194.963 |
| 1988 | 18150 | 16378.87 | 1771.132 | 1771.132 |
| 1989 | 18970 | 17457.57 | 1512.434 | 1512.434 |
| 1990 | 19328 | 18402.04 | 925.9607 | 925.9607 |
| 1991 | 19337 | 18998.99 | 338.0126 | 338.0126 |
| 1992 | 18876 | 19239.96 | 363.9582 | -363.958 |
| Tracking Signal | | | 0.570695 | |

Table 4. The results of the MA and WMA (weight: 0.1,0.2,0.3,0.4) forecasting models for dataset 1

| | MA forecasting model | WMA forecasting model |
|---|---|---|
| MAPE | **6.37%** | **5.27%** |
| MAD | 1078.85 | 893.01 |
| RSFE | 12643.75 | 9783.00 |
| Tracking signal | **11.72** | **10.96** |

Table 5. The results of the OWA-MA forecasting model for dataset 1

| | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|
| MAPE | **6.37%** | 16.91% | 12.29% | 4.35% | 3.63% | **3.01%** |
| MAD | 1078.85 | 2855.58 | 2080.53 | 736.05 | 613.25 | 504.22 |
| RSFE | 12643.75 | 51400.38 | -14646.75 | 420.06 | 323.74 | 232.22 |
| Tracking signal | **11.72** | 18.00 | -7.04 | 0.57 | 0.53 | **0.46** |

**4.2 The dataset of product demand**

This study adopts the forecast demand table. The database contains a total of 30 periods as listed in Table 6.

Table 6. The dataset of product demand

| Week | Demand | Week | Demand |
|---|---|---|---|
| 1 | 800 | 16 | 1700 |
| 2 | 1400 | 17 | 1800 |
| 3 | 1000 | 18 | 2200 |
| 4 | 1500 | 19 | 1900 |
| 5 | 1500 | 20 | 2400 |
| 6 | 1300 | 21 | 2400 |
| 7 | 1800 | 22 | 2600 |
| 8 | 1700 | 23 | 2000 |
| 9 | 1300 | 24 | 2500 |
| 10 | 1700 | 25 | 2600 |
| 11 | 1700 | 26 | 2200 |
| 12 | 1500 | 27 | 2200 |
| 13 | 2300 | 28 | 2500 |
| 14 | 2300 | 29 | 2400 |
| 15 | 2000 | 30 | 2100 |

Finally, the paper compares results of the MA and WMA forecasting models shown in Table 7 and Table 8.

Table 7. The results of the MA and WMA (weight: 0.1,0.2,0.3,0.4) forecasting models for dataset 2

| | MA forecasting model | WMA forecasting model |
|---|---|---|
| MAPE | **12.35%** | **12.75%** |
| MAD | 248.08 | 255.00 |
| RSFE | 2550.00 | 1910.00 |
| Tracking signal | **10.28** | **7.49** |

Table 8. The results of the OWA-MA forecasting model for dataset 2

| | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|
| MAPE | 12.35% | 17.69% | 14.72% | **13.06%** | **13.25%** | **13.64%** |
| MAD | 248.08 | 372.46 | 309.45 | 259.50 | 262.56 | 269.23 |
| RSFE | 2550.00 | 9108.77 | 6641.38 | 1400.53 | 1001.44 | 600.00 |
| Tracking signal | 10.28 | 24.46 | 21.46 | 5.40 | 3.81 | **2.23** |

Then, we compute tracking signal to evaluate the estimated accuracy. The tracking signal is more closer zero is better. In Table 7 and Table 8, we can see that the tracking signal of the proposed approach with OWA operator is closer zero, that is, the proposed model is more robust than MA and WMA.

**5. Conclusions**

In this paper, we have proposed an OWA-MA forecasting model for estimate time series problem. From the result, we can see that the tracking signal of the proposed approach is better than the existing methods. The proposed OWA-MA forecasting model can get robust estimated accuracy than the existing methods, that is, the estimated accuracy rate of the proposed method is better than other methods. Therefore, we can use OWA-MA forecasting model to increase estimating accuracy.

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