行政院國家科學委員會專題研究計畫 成果報告

一般化誤差分配下之分位數法風險值計算
研究成果報告(精簡版)

計 畫 類 別 ： 個別型
計 畫 編 號 ： NSC 96-2416-H-343-006-
執 行 期 間 ： 96 年 08 月 01 日至 97 年 07 月 31 日
執 行 單 位 ： 南華大學財務金融學系

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處 理 方 式 ： 本計畫涉及專利或其他智慧財產權，2 年後可公開查詢

中 華 民 國 97 年 08 月 11 日

Value-at-Risk Based on Generalized Error Distribution Using a Quantile Approach

Abstract: This study proposes the quantile method under the Generalized Error Distribution (GED) VaR forecasts with non-normality assumption, fitting the returns data with the GED. This application is more efficient and flexible not only for accommodating both normality and non-normality situations in one model, but also for retaining the easy usages characteristic of the variance-covariance. In the light of results of the failure rates and Kupiec test, the empirical result shows that the proposed method can considerably enhance the estimation accuracy of Value-at-Risk.
Keywords **：*Value at Risk, Variance-covariance, Quantile***

1. Introduction

Value-at-Risk (VaR) has recently become the most important benchmark for determining risks in portfolios. According to Jorin (2000), VaR summarizes the expected maximum loss over a target horizon with a specified level of confidence. Importantly, VaR can aggregate all risks in a portfolio into a single number. Three major approaches, namely historical simulation, variance-covariance, and Monte Carlo simulation, have been widely adopted in practice to estimate VaR precisely. The variance-covariance method derives the VaR exactly on the basis of the distributional assumptions. Hence, the variance-covariance brings two major advantages for VaR estimation. First, if the normal distribution is assumed, then an analytic solution can be obtained for VaR standards with a holding period of greater than one day. Second, the model parameters are generally easy to determine in the normal distribution. The variance-covariance approach is also computationally fast, even with a very large number of assets, since it replaces each position by its linear exposure.

Although the superiority of variance-covariance over its parametric counterparts for the above reasons, it still has a major limitation that must be recognized and overcome. The conditional distribution of short horizon financial assets returns is leptokurtic, with tails that are fatter than those of normal distribution. This phenomenon has been extensively investigated by Koedijk *et al*. (1992), Pictet *et al*. (1996) and Huisman *et al*. (1998). In this situation, a model based on a normal distribution would underestimate the proportion of outliers, thus underestimating the true VaR.

To maintain the privileged merits of variance-covariance, this investigation proposes the quantile method under the Generalized Error Distribution (GED) VaR forecasts with non-normality assumption, fitting the returns data with the GED. This application is more efficient and flexible not only for accommodating both normality and non-normality situations in one model, but also for capturing various fat-tailed distributions other than the variance-covariance. This feature of the proposed application is particularly helpful, since it enables extrapolation of the optimal tail-fatness by real data. Therefore, the optimal GED model can be selected to fit returns data, and capture time-varying fatness and volatilities.

The remainder of the paper is organized as follows. Section 2 describes the research method. Section 3 then analyzes the empirical evidence on the forecasting accuracy of our proposed method. The performance of the proposed method is compared with that of the variance-covariance method by implementing the Kupiec (1995) test and failure ratios on five stock indices. Analytical results demonstrate that the proposed method is much more stable than the variance-covariance. Conclusions are finally drawn in Section 4.

2. Research Method

Successfully implementing VaR depends strongly on the ability to accurately estimate the conditional distribution of asset returns. This study considers two possible distributions. The first is the normal distribution. The most important reason underlying this choice is that VaR calculations almost always assume a normal distribution, mainly because of its practical advantages particularly for models with cointegration (Johansen, 1988).

The second distribution reviewed in this study is the GED. Despite the practical advantages of normal distribution, its assumptions are rarely fulfilled in daily financial data. This investigation employs the GED model, which is frequently employed to modeling financial assets returns with non-normal distribution in literature (e.g., Wei (1998), Koutmos (1999)). Additionally, data in this study indicate that the real distribution was more thick-tailed than the normal distribution in all cases (see Table 1). This application is more efficient and flexible not only for accommodating both normality and non-normality situations in one model, but also for capturing various non-normal fat-tailed distributions and, thus, seems to correspond more closely to reality.

The following subsections discuss in detail the variance-covariance approach, and the proposed estimation method.

2.1 The Variance-Covariance and Quantile Method under GED

Variance-covariance approaches assume that the market returns have a normal distribution, as follows:

$$VaR_t = \mu_t + \sigma_t Z(\alpha) \tag{3}$$

In the above, $Z(\alpha)$ denotes the $\alpha\%$ quantile of the standard normal distribution, and μ_t and σ_t^2 denote the mean and variance, respectively. The VaR at time *t* is usually determined with the forecasts of μ_t and σ_t^2.

Conventionally, a portfolio's VaR is calculated by the variance-covariance approach assuming of conditional normal returns. However, log returns are frequently found not to be normally distributed (see Boudoukh *et al.*(1997) and Hull and White(1998)). To preserve the feature of convenient manipulation of the variance-covariance, this study uses the quantile method under GED VaR forecasts.

The probability density function of the GED is given by:

$$f(R) = \frac{v \exp\left[-\left(\frac{1}{2}\right)\left|\frac{R}{\omega\sigma}\right|^{v}\right]}{\omega 2^{(1+1/v)}\Gamma(1/v)\sigma} \quad , \quad 0 < v < \infty \tag{4}$$

Where *R* denotes the rate of return; σ denotes the standard deviation; $\Gamma(v^{-1})$ denotes a standard gamma function, and $\omega \equiv \left[2^{(-2/v)}\Gamma(1/v)/\Gamma(3/v)\right]^{1/2}$, where *v* denotes a tail-fatness parameter. When v=2, R represents a standard normal distribution. The distribution of R has thin tails when $v>2$, and fat tails when for $v<2$ (e.g., R has a double exponential distribution when v=1). A smaller (larger) v is leads to a fatter (thinner) tail.

Nevertheless, using GED for estimation involves determining the value of v in advance, which in turn shapes the distribution of returns. Guermat and Harris (2002) fitted returns data only by several designated values of v. In contrast, this study suggests relying on kurtosis as guiding light to search out the optimal value of v. With the distribution assumption of GED, the kurtosis (k) and v can be equated by:

$$k = \frac{\Gamma(1/v)\Gamma(5/v)}{[\Gamma(3/v)]^2} \tag{5}$$

In other words, the kurtosis (k) of accumulated returns data collected is calculated, and then plugged into Eq. (5) to backward solve the v. Moreover, the time-varying values of v may also be calculated through moving windows. Therefore, the proposed approach not only considers the situation of time-varying returns distributions, but also determines the conditional distributions.

2.2 Model Evaluation

The accuracy performance of proposed models was measured by the widely used Kupiec (1995) test and failure ratios. In the performance comparison of different models, this study uses one day as the specific period for the assessment of VaR at 99%, 95%, and 90% levels of confidence. The principles of using these tests for evaluating the accuracy of proposed models are then explained. The use of the Kupiec test method is described as follows:

2.2.1 Kupiec test method

The unconditional coverage of the Kupiec(1995) test is a LR_{PF}, shown as equation (6), based on the binomial distribution, and is used to determine if the failure ratio is compatible with the expected level of confidence. The sample size is T, and the frequency of failure is the binomial probability of x, $\binom{T}{x}(1-c)^{T-x}c^{x}$, under which the assessed value of risk must have an unconditional coverage ratio c equal to the expected level of c_0. In other words, the null hypothesis $H_0: c = c_0$, and the

LR_{PF} is the distribution of χ^2 with degrees of freedom equal to 1.

$$LR_{PF} = -2\ln\left[(1-c_0)^{T-x} c_0^{\ x}\right] + 2\ln\left[(1-(x/T))^{T-x}(x/T)^{x}\right] \tag{6}$$

2.2.2 Binary Loss Function (BLF)

This study also adopts the BLF as an indicator for the accuracy of the models. The BLF is based on the concept of failure ratios, whereby an actual loss greater than the VaR value is considered as failure. For each failure, a constant of 1 is assigned; otherwise it is zero. If a VaR model truly provides the level of coverage defined by its confidence level, then the average BLF over the full sample equals 0.05 for the 95th percentile VaR, and 0.01 for the 99th percentile VaR. Therefore, a BLF value closer to the confidence level from the model indicates a higher accuracy is.

$$BLF_{i,t+1} = \begin{cases} 1 \ \text{ if } \ \Delta P_{i,t+1} < VaR_{i,t} \\ 0 \ \text{ if } \ \Delta P_{i,t+1} \geq VaR_{i,t} \end{cases} \tag{7}$$

3. Empirical Study

3.1 Sources and data analysis

The model was tested by the daily data of five international indices (S&P 500, FTSE 100, DAX, Nikkei 225, and TAIEX) from January 1, 1990 to December 31, 2006. Continuously compounded returns were then calculated as the first difference of the natural logarithm of each series, $R_t = \ln(P_t / P_{t-1})$, where R_t denotes the return index value for date *t*.

Table 1 shows test results from the assumption that each return index series is normally distributed. The statistical values in Table 1 show that the average daily returns are about 0. This result is consistent with the results of previous studies about the long-term average daily return rate of stock market. Table 1 also shows the extreme values of the entire sample pool; that is, the mean plus (minus) three times the standard deviation, and at the 1% and 99% percentile. The corresponding critical values at 1% and 99% percentile, calculated by adding (subtracting) the mean with three times the standard deviation, indicate that significant deviation was found between the lowest (highest) extreme values and the mean plus (minus) three times the standard deviation. A similar phenomenon occurred between the lowest (highest) extreme values and the critical value at 1% (99%) percentile. This finding implies that the tail of the actual distribution would be thicker than that of the normal distribution. Additionally, the skewness and kurtosis coefficients are different from those under normal distribution. The findings from the above measurement and verification demonstrate that the returns on stock markets are not normally distributed, and are leptokurtic. The assumption of normal distribution cannot fully reflect the fat tail in a leptokurtic curve of distribution.

3.2 The empirical findings and analysis

3.2.1 The results of Kupiec tests

With the confidence levels of 99%, 95%, and 90%, the proposed method is evaluated against the variance-covariance on the forecasting capability of one-day VaRs of five international indices. Tables 2 reports the results, evaluated by the Kupiec (1995) test, of whether various proposed models can provide precise portfolio VaR estimates.

The Kupiec test of variance-covariance was higher than the nominal significance level of 1% at the 99% VaR confidence level for all four indices. Only for the Nikkei 225 series is not rejected. The quantile method under GED VaR forecasts was found to perform significantly better than the variance-covariance at the 99% VaR confidence level.

At the 95% VaR confidence level, the null hypothesis of variance-covariance was rejected at the 1% level for the S&P 500 and at the 5% level for the TAIEX. The null hypothesis of the Kupiec test was rejected only for S&P 500 series in the proposed model, and then only at the 1% level.

Table 2 reports the results for the 90% VaR confidence level. The VaR estimates produced by the variance-covariance strongly rejected the null hypothesis at the 1% level for any indices, as in Table 1. Using the quantile method under GED VaR forecasts improved the unconditional coverage in all cases. The null hypothesis of the quantile method under GED VaR forecasts was rejected for only two indices, and only at the 5% level.

The results in Table 2 demonstrate that the variance-covariance cannot provide promisingly accurate VaR estimates for the five international indices. Therefore, the overall test results of the variance-covariance show that this method cannot promise its stable forecasting capability on accuracy. Conversely, the quantile method under GED VaR forecasts produced very accurate forecasts in nearly all cases.

3.2.2 The results of BLF tests

Since the BLF concentrates on the concept of failure ratio, it can also be used as an indicator for the accuracy of estimative models. Therefore, the closer the BLF value closer to the specified confidence level shows a more accurate model. Table 3 indicates that most of the BLF values of the proposed models were closer than those of the variance-covariance to the specified significance levels. The excellent performance of the proposed approaches on the failure ratio tests also demonstrates their strong capability of capturing the tail behavior of returns. Therefore, the quantile method under GED VaR is not only easy to use, but also more accurate than the variance-covariance.

4. Conclusion

Despite its ability to accurately estimate VaR, variance-covariance still must deal with inherent problems resulting from using the normal distribution. The resulting negative impact on the estimation accuracy of the portfolio VaR can make the variance-covariance unusable. Additionally, the returns of assets pricing follow a GED more closely than a normal distribution. To solve these problems, this study suggests using the quantile method under GED VaR forecasts. The proposed method has the following two major advantages: (1) the GED is more efficient and flexible in accommodating various fat-tailed distributions and (2) the extended applications of GED are easy to implement, and still enable convenient manipulation of the variance-covariance.

Most interestingly, the capturing capability of the proposed method is derived from the dynamic availability of tail-fatness obtained by calculating the kurtosis. Results of this study demonstrate that the proposed method indeed has a better forecasting accuracy than the variance-covariance for portfolio VaR.

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TABLE 1

The Descriptive Statistics of Returns of the Five Stock Indices

	S&P 500	FTSE 100	DAX	Nikkei 225	TAIEX
Mean (A)	0.00032	0.00022	0.00026	-0.00019	-0.00005
Standard Deviation (B)	0.00999	0.01041	0.01474	0.01479	0.01799
Kurtosis	3.98931	7.52688	11.26111	3.07969	2.46542
Skewness	-0.08591	0.04776	-0.27212	0.15936	-0.20269
Lowest	-0.07113	-0.09160	-0.15803	-0.07234	-0.07045
A-3B	-0.02965	-0.03101	-0.04397	-0.04455	-0.05401
1% quartile	-0.02625	-0.02947	-0.04315	-0.03873	-0.05943
99% quartile	0.02760	0.02696	0.03901	0.03751	0.05169
A+3B	0.03029	0.03145	0.04449	0.04417	0.05391
Highest	0.05574	0.11554	0.14810	0.12430	0.06577
Sum	4292	4280	4277	4181	4582

TABLE 2

The Kupiec Tests Results of Competing Models

S&P 500	**C=0.01**	**C=0.05**	**C=0.1**
ND	6.82**	7.06**	20.76**
GED	0.09	7.49**	1.19
FTSE 100	**C=0.01**	**C=0.05**	**C=0.1**
ND	29.02**	2.07	21.01**
GED	0.27	1.46	1.74
DAX	**C=0.01**	**C=0.05**	**C=0.1**
ND	27.42**	0.25	10.43**
GED	0.27	1.83	4.18*
Nikkei 225	**C=0.01**	**C=0.05**	**C=0.1**
ND	2.62	1.33	7.62**
GED	0.66	1.51	0.06
TAIEX	**C=0.01**	**C=0.05**	**C=0.1**
ND	13.22**	5.08*	39.73**
GED	1.21	3.58	5.97*

Note: C=0.01 stands for the confident level of 99%. C=0.05 stands for the confident level of 95%. C=0.1 stands for the confident level of 90%. ND stands for Normal distribution. GED stands for Generalized Error Distribution. *,** denote significance at the 5%, and 1% levels.

TABLE 3

The Failure Ratios of Competing Models

S&P 500	**C=0.01**		**C=0.05**		**C=0.1**	
	(Theoretic NF: 38)		(Theoretic NF: 190)		(Theoretic NF: 379)	
	Total NF	FR	Total NF	FR	Total NF	FR
ND	**55**	**0.015**	**155**	**0.041**	**295**	**0.078**
GED	**36**	**0.009**	**154**	**0.041**	**348**	**0.092**
FTSE 100	**C=0.01**		**C=0.05**		**C=0.1**	
	(Theoretic NF: 38)		(Theoretic NF: 189)		(Theoretic NF: 378)	
	Total NF	FR	Total NF	FR	Total NF	FR
ND	**73**	**0.019**	**170**	**0.045**	**289**	**0.076**
GED	**41**	**0.011**	**173**	**0.046**	**343**	**0.091**
DAX	**C=0.01**		**C=0.05**		**C=0.1**	
	(Theoretic NF: 38)		(Theoretic NF: 189)		(Theoretic NF: 378)	
	Total NF	FR	Total NF	FR	Total NF	FR
ND	**74**	**0.019**	**181**	**0.048**	**312**	**0.082**
GED	**41**	**0.011**	**171**	**0.045**	**364**	**0.096**
Nikkei 225	**C=0.01**		**C=0.05**		**C=0.1**	
	(Theoretic NF: 37)		(Theoretic NF: 184)		(Theoretic NF: 368)	
	Total NF	FR	Total NF	FR	Total NF	FR
ND	**47**	**0.013**	**169**	**0.046**	**317**	**0.086**
GED	**32**	**0.009**	**168**	**0.046**	**353**	**0.096**
TAIEX	**C=0.01**		**C=0.05**		**C=0.1**	
	(Theoretic NF: 41)		(Theoretic NF: 204)		(Theoretic NF: 408)	
	Total NF	FR	Total NF	FR	Total NF	FR
ND	**66**	**0.016**	**172**	**0.042**	**288**	**0.071**
GED	**48**	**0.012**	**176**	**0.043**	**339**	**0.083**

Note: NF denotes number of failure. FR denotes failure ratio