行銷市場中的價格避險理論

Price Hedge Theory in Marketing

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Abstract

Pricing is a problem when a firm has to set a price for the first time. This happens when the firm develops or acquires a new product, when it introduces its regular product into a new distribution or geographical area, and when it enters bids on new contract work. Many companies try to set the price that will maximize current profits. They estimate the demand and costs associated with alternative prices and choose the price that produces maximum current profit, cash flow, or rate of return on investment.

There are some problems associated with current profit maximization. It assumes that the firm has knowledge of its demand and cost functions; in reality, the demand is difficult to estimate and unpredictable. Due to its unpredictability, we assume that the demand follows a lognormal random walk. We then develop a mathematical modeling of pricing processes by stochastic calculus, which is just like the mathematical modeling of financial processes. From Ito's lemma, the profit of a product has a correlation with the demand, is also unpredictable and follows a random walk. Such a random behavior is the risk of marketing. By choosing a price strategy to eliminate the randomness, which is called price hedging, we obtain a risk-free profit determined by the Black-Scholes equation. This riskless profit, which is predictable, is the same as the growth we get if we put the equivalent amount of cash in a risk-free interest-bearing account.

***Keywords*: Price hedging, Randomness, Risk**

1. Introduction

Pricing is an important tool in the delivery of human services. The price of a service is an integral part of that service. Price can be viewed as a statement of value, a reflection of costs, or a marketing strategy. Traditional approaches to pricing have been either cost or market oriented. The former is supply focused in that price is viewed as a reflection of the cost of the input used to create the goods or services, whereas the latter is demand focused in that price is a tool for eliciting a certain response from a potential consumer.

Economic theory relates price to cost, competition, and the elasticity of demand. In

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competitive markets the combined forces of competition and the desire of sellers to maximize profits will lead sellers to produce to the point at which price, marginal cost, and average cost are equal (McCain, 1981). From a marketing perspective, price is a tool for pursuing the goals of an organization (Gabor, 1988). Traditional business practices focus on maximizing profits. Therefore, price is a strategic tool for eliciting the demand of consumers and enhancing the image of a product or service.

Price is the only item in the marketing mix that generates revenue. The other place, product, and promotion all generate cost. Low prices may generate demand; however, higher prices make outstanding performance possible. In addition, higher prices may lead consumers to view services as being of higher quality.

The stage in the life cycle of a service product may be an important feature of pricing. New services or products that are functionally unique face little competition. Sellers may offer such products at a price that is much higher than the cost of production, thus allowing the company to maximize its profit. Another approach is to offer a new service or product at or near the cost of production, thus allowing the seller to penetrate the market and discouraging competitors from entering the market. The purpose of this approach is still to maximize the seller’s profit. No matter what approach, there are some problems associated with current profit maximization. To perform “optimizing behavior” the agents must know each other’s demand and supply schedules and then agree to adjust their prices to produce clearing. Savings, cash and financial markets are irrelevant here because no agent needs to set aside cash for an uncertain future (McCauley, 2004). In reality, the demand is difficult to estimate and unpredictable. Real markets made up of qualitatively different kinds of agents with real desires and severe limitations on the availability of information and the ability to sort and correctly interpret information. Markets are full of surprise and unpredictability.

Unpredictability is the basic property of markets. Unpredictability creates risk. Businesses confront numerous risks, and they have choices in managing them. The task of defining a menu of realistic choices and specifying their benefits and costs is complex. A variety of ways analyzing risk exist (Boehlje and Lins, 1998). Various risk-management strategies can be justified based on theoretical analyses, and many corporations employ some financial tools to manage risks (Bodnar et al. 1998). Yet academics know little about corporate risk-management practices and how they relate to theory. Our knowledge about sales’ risk-management practices also is incomplete, but considerable research has been conducted on the theory and practice of price risk management by agricultural firms (Tomek and Peterson 2001).

The price risk to be managed is that associated with the cash market, and an understanding of cash price behavior is a critical element of risk management. The characteristics of commodity prices are well documented, but this has not made risk management easy. A typical price series, whatever the frequency, exhibits considerable variability and positive autocorrelation. There are occasional prices jumping abruptly to a high level relative to its long-run average. Thus the distribution of observed prices is skewed to the right and, in many cases, displays substantial kurtosis (Deaton and Laroque, 1992). Price changes are nonlinearly dependent and higher moments are correlated (Yang and Brorsen 1992). Given the complexity of these time-series features, modeling commodity prices has been an impossible task. Economists have not reached a consensus about the best model for commodity prices.

Fundamentally, commodity price behavior over time is a mixture of systematic intra- and inter-year fluctuations plus randomness, and the variability of prices depends on information flows regarding supply and demand. Cash prices may be considered to be mean reverting to some long-run average or trend. Trends generally are associated with general inflation and deflation in the economy, changes in consumer preferences, increases in population and income, and technological changes in production. If marginal costs of production decline over the years, the price level for that commodity will trend downward.

For effective risk management, understanding the mean price level is not sufficient, since it is the extreme prices, which occur with low probability that can bankrupt firms. If economists could characterize fully the systematic component of commodity price behavior and produce relatively accurate price forecasts, the price-risk-management techniques could deal with random deviations of cash prices around the known pattern of the mean. Despite voluminous efforts in commodity price analysis, however, it has proven to be very difficult to obtain good forecasts of commodity prices. Hence, much of price variability can be classified as risk.

To avoid the price risk, financial products, such as futures or options etc., are invented to reduce or hedge the risk. Futures contracts are used primarily to hedge storage, merchandising, or production decisions, and an understanding of basis relationships and basis risk are important for hedging effectiveness. An understanding of the probability distributions of futures prices is important to decision makers. Optimal hedges in futures depend on the parameters of the underlying probability distributions, and the estimates of these parameters depend, in turn, on the analyst's assumed model of the distribution (McNew and Fackler, 1994). Since futures options require the delivery of an underlying futures contract when exercised, basis risk exists, but options contracts offer an alternative risk-management mechanism.

The model for pricing options on assets by Black and Scholes (1973) is based on the underlying assumption that the asset price, S, follows geometric Brownian motion: $dS/S = \mu\, dt + \sigma\, dW$, where μ is the expected growth rate in S, σ is its volatility, and dW is a Wiener process. Although the validity of this model is debatable, volatility is one of the critical factors determining option premiums. Volatility can be viewed in two ways: historical volatility is the variation in the underlying asset price, which can be estimated ex post; and implied volatility is a value that equates the market premium with the theoretical premium. The latter, which can be estimated from the Black-Scholes model (Black and Scholes 1973) given the interest rate, strike price, market price of the underlying asset, and time to maturity, is regarded as a forecast.

Option markets, like futures markets, generally are thought to be efficient. There could be times when the market premium is under- or overvalued, but such discrepancies are thought to be arbitraged quickly away (efficient market hypothesis). The idea of the efficient market hypothesis is based on the fact that it is very difficult in practice to beat the market. There are no systematically repeated patterns in the market, which is a risk. Although the risk is unknown and unpredictable, we can hedge the risk with options. Black and Scholes (1973) demonstrate that it is possible to create a riskless hedge by forming a portfolio containing stock and European call options. The sources of change in the value of the portfolio must be prices, since at a point in time the quantities of the assets are fixed. If the call price is a function of the stock price and the time to maturity, then changes in the call price can be expressed as a function of the changes in the stock price and changes in the time to maturity of the option. Black and Scholes then observe that at any point in time the portfolio can be made into a riskless hedge by choosing an appropriate mixture of stock and calls, e.g., if the hedge portfolio is established with a long position in the stock and a short position in the European call and if the stock price rises, then the increase in the value of the portfolio from the profit on the long position in the stock is offset by the decrease in the value of the portfolio from the loss which the increase in the stock price generates through the short position on the option, and vice versa. If quantities of the stock and option in the hedge portfolio are continuously adjusted in the appropriate manner as the asset prices change over time, then the return to the hedge portfolio becomes riskless. Therefore, the portfolio must earn the riskless rate.

Let $\Pi(S, t)$ to denote the value of a portfolio of a short option position and one long position in some quantity Δ, delta, of the underlying asset:

$$\Pi(S,\ t) = V(S,\ t) - \Delta S, \tag{1}$$

where $V(S, t)$ is the option value and S is the asset price. It assumed that the motion of the asset price, S, can be described by Geometric Brownian motion (Samuelson, 1965):

$$dS/S = \mu\ dt + \sigma\ dW, \tag{2}$$

then Ito's lemma (Malliarist, 1983) can be employed to express $dV(S, t)$ as

$$dV(S,t) = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial^2 S} dt. \tag{3}$$

Substituting Eq. (3) for $dV(S, t)$ in Eq. (1) yields

$$d\Pi(S,\ t) = dV(S,\ t) - \Delta dS = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial^2 S} dt - \Delta dS. \tag{4}$$

Note that the only stochastic term in the expression for $d\Pi(S, t)$ is dS. The rest are deterministic. The dS random terms are the risk in our portfolio. Is there any way to reduce or even eliminate this risk? This can be done in theory by choosing $\Delta(S,t) = \partial V/\partial S$, and then the randomness is reduced to zero. Any reduction in randomness is generally termed hedging. This perfect elimination of risk, by exploiting correlation between two instruments is generally called delta hedging (Wilmott, 1998).

After choosing $\Delta(S,t) = \partial V/\partial S$, we hold a portfolio whose value changes by the amount

$$d\Pi(S,\ t) = \frac{\partial V}{\partial t}dt + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial^2 S}dt \tag{5}$$

This change is completely riskless. If we have a completely risk-free change $d\Pi(S,\ t)$ in the portfolio value $\Pi(S,\ t)$ then it must be the same as the growth we would get if we put the equivalent amount of cash in a risk-free interest-bearing account:

$$d\Pi(S,\ t) = r\ \Pi(S,\ t)\ dt = r\big(V(S,\ t)\ - \Delta S\big)dt = r\big(V(S,\ t)\ - S\,\partial V/\partial S\big)dt\,, \tag{6}$$

where r is the interest rate. This is an example of the no arbitrage principle. Substituting Eq. (5) into Eq. (6) we find that

$$r\big(V(S,\ t)\ - S\,\partial V/\partial S\big)dt = \frac{\partial V}{\partial t}dt + \frac{1}{2}\frac{\partial^2 V}{\partial^2 S}\sigma^2 S^2 dt\,. \tag{7}$$

On dividing by dt and rearranging Eq. (7) defines a Black-Scholes differential equation (Black & Scholes, 1973) for the value of the option,

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial^2 S} + rS\frac{\partial V}{\partial S} - rV = 0\,, \tag{8}$$

subject to the boundary condition and that at the terminal date, the option price must be equal to the maximum of either the difference between the stock price and the exercise price or zero. The differential equation (8) can be solved for the call price. Black and Scholes transform the equation into the heat exchange equation from physics to find the solution.

The freedom of choosing a portfolio to eliminate the risk is important to the risk management. From a risk-free portfolio, we can estimate how much risk of choosing another different portfolio. There is also such freedom in marketing. A company has the freedom to adjust the price of the product or service given by the company. This price adjustment freedom can eliminate the randomness of the profits of a product. Without the randomness and risk, the profits of a product are deterministic. The riskless profits must be the same as the growth we would get if we put the equivalent amount of cash in a risk-free interest-bearing account. We show that the very basic price of a product or service is determined by the risk-free profits. In Section 2, we show how the freedom to adjust the price can eliminate the randomness of profits. The Black-Scholes equation of profits is derived. The price of a product is determined by the Black-Scholes differential equation. Conclusions are given in Section 3.

2. Price hedging

How are prices set? Through most of history, buyers and sellers negotiating with each other set prices. Sellers would ask for a higher price than they expected to receive, and buyer would offer less than they expected to pay. Through bargaining, they would arrive at an acceptable price.

Price is the only element in the marketing mix that produces revenue; the other elements produce costs. Pricing and price competition was rated as the important problem facing marketing executives. Yet many companies do not handle pricing well. The most common mistakes are these: Pricing is too cost oriented; price is not revised often enough to capitalize on market changes; and price is set independent of the rest of the marketing mix rather than as an intrinsic element of market-positioning strategy.

Many companies try to set the price that will maximize current profits. They estimate the demand and costs associated with alternative prices and choose the price that produces maximum current profit, cash flow, or rate of return on investment.

There are some problems associated with current profit maximization. It assumes that the firm has knowledge of its demand and cost functions; in reality, the demand is difficult to estimate and unpredictable. Also, the effects of other marketing-mixing variables and competitors' reactions are unknown to the company. Due to its unpredictability, the demand of a product or service is randomized. No known pattern of consumers is predictable.

From its randomness, we assume that the demand follows a lognormal random walk. Geometric Brownian motion can describe the motion of the demand quantity, Q:

$$dQ/Q = \mu\, dt + \sigma\, dW, \tag{9}$$

where μ is the expected growth rate in Q of a product, σ is its volatility, and dW is a Wiener process. Assume that the profit $F(Q, t)$ and cost $C(Q, t)$ of a product are the functions of the demand quantity Q and time t. Then the profit is the difference between total revenue and total cost:

$$F(Q,\ t) \ = \ PQ - C(Q,\ t), \tag{10}$$

where P is the price of a product. We can rearrange Eq. (10) and write the equation into a differential form

$$dC(Q,\ t) \ = \ P\ dQ - dF(Q,\ t). \tag{11}$$

Ito's lemma (Malliarist, 1983) can be employed to express $dF(Q, t)$ as

$$dF(Q,t) = \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial S} dQ + \frac{1}{2}\sigma^2 Q^2 \frac{\partial^2 F}{\partial^2 Q} dt\ . \tag{12}$$

Substituting Eq. (12) for $dF(Q, t)$ in Eq. (11) yields

$$dC(Q,t) = P\ dQ - \left(\frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial S} dQ + \frac{1}{2}\sigma^2 Q^2 \frac{\partial^2 F}{\partial^2 Q} dt \right). \tag{13}$$

The only stochastic term in Eq. (13) is dQ. The rest are deterministic. The dQ random terms are the risk in our revenue. From the freedom of the price adjustment of a company, this risk can be eliminated by choosing $P(Q,t)=\partial F/\partial Q$, and then the randomness is reduced to zero. This reduction in randomness is termed price hedging and completely riskless. If we have a completely risk-free change $dC(Q, t)$ in the cost $C(Q, t)$ then it must be the same as the growth we would get if we put the equivalent amount of cash in a risk-free interest-bearing account:

$$dC(Q,\ t)=r\ C(Q,\ t)\ dt=r\left(PQ-F(Q,\ t)\right)dt=r\left(Q\,\partial F/\partial S-F(Q,\ t)\ \right)dt, \tag{14}$$

where r is the interest rate. This is an example of the no arbitrage principle. Substituting Eq. (13) into Eq. (14) we find that

$$r\left(Q\,\partial F/\partial S-F(Q,\ t)\ \right)dt=-\frac{\partial F}{\partial t}dt-\frac{1}{2}\sigma^2Q^2\frac{\partial^2 F}{\partial^2 Q}dt\,. \tag{15}$$

On dividing by dt and rearranging Eq. (15) we get

$$\frac{\partial F}{\partial t}+\frac{1}{2}\sigma^2Q^2\frac{\partial^2 F}{\partial^2 Q}+rQ\frac{\partial F}{\partial Q}-rF=0\,. \tag{16}$$

We have derived the Black-Scholes equation for the riskless profit of a product. The differential equation (16) can be solved for the riskless profit. We can also transform the equation into the heat exchange equation from physics to find the solution.

To obtain $F(Q, t)$, we have to solve the differential equation (16) with the boundary conditions and the initial condition. From $P(Q,t)=\partial F/\partial Q$, we then determine the price of a product. Price hedging is an example of a dynamic hedging strategy. From one time step to the next the quantity $P(Q,t)=\partial F/\partial Q$ changes since it is, like $F(Q, t)$, a function of the ever-changing variables Q and t. Because it is riskless, the price determined by price hedging is the very basic price of a product. From price hedging, we can determine how much risk of choosing a different price strategy. Risk management becomes possible in marketing.

3. Conclusions

We study the pricing problem when a firm has to set a price for the first time. This happens when the firm develops or acquires a new product, when it introduces its regular product into a new distribution or geographical area, and when it enters bids on new contract work. Many companies try to set the price that will maximize current profits. They estimate the demand and costs associated with alternative prices and choose the price that produces maximum current profit, cash flow, or rate of return on investment.

We show that there are some problems associated with current profit maximization. The firm has no knowledge of its demand and cost functions; in reality, the demand is difficult to estimate and unpredictable. Due to its unpredictability, we assume that the demand follows a lognormal random walk. Then, the profit of a product or service, which is a function of the demand and time, is also a random walk. This random process is mathematically modeled by stochastic calculus, which is just like the mathematical modeling of financial processes. From Ito's lemma, the profit of a product also follows a lognormal random walk. We termed such a random behavior being the risk of marketing. By choosing a price strategy to eliminate the randomness, we obtain a risk-free profit determined by the Black-Scholes equation. This riskless profit, which is predictable, is the same as the growth we get if we put the equivalent amount of cash in a risk-free interest-bearing account.

From pricing hedging and the Black-Scholes equation, we determined the very basic price of a product, which is changing with time and the demand. Such a dynamical price can revise often enough to capitalize on market changes. Risk management in marketing is tied with our price hedge theory.

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