二元公式化架構的應用－啟發法在動態批量模式中的比較

An Application of Binary Formulation Framework for Comparing the Heuristics in Dynamic Lot-sizing Model

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摘　要

本研究透過二元公式化架構將複雜的動態批量模式具體化，呈現可行的批量生產政策與散佈趨勢。在界定明確的需求範例下，最小成本曲線與最大成本曲線可經由二元公式化架構清楚的描繪。此外，本研究利用一些重要的啟發法與指標，去反映和識別敏感性區間。在需求期間不確定的情況下，管理者可以訂定適當的敏感性區間範圍，選擇適合的啟發法，簡化批量生產政策的決策程序。

關鍵詞：批量、動態規劃、二元公式化、啟發法

Abstract

This study visualizes the complicated dynamic lot-sizing (DLS) model to present the scattered trend of feasible production policies through the binary formulation framework. The minimum total costs curve and maximum total costs curve are plotted visibly under the demand patterns with well-defined termination point. Moreover, some of the significant heuristics are used to reflect the sensitivity interval through all demand patterns and the appropriate range of sensitivity interval could be recognized by means of several indices. Managers could decide the appropriate range of sensitivity interval and select an appropriate heuristics to simplify the processes of lot-sizing decisions.

Keywords：Lot Sizing, Dynamic Programming, Binary Formulation, Heuristics.

1. Introduction

The dynamic lot-sizing (DLS) models have been a significant procedure to determine the optimal timing and amount for production policies in practice. Wagner and Whitin (1958) planning and inventory control, but the disadvantage of Wagner-Whitin (W-W) algorithm suffered from a computational complexity. Zangwill (1969) and Love (1972) developed efficient dynamic programming based algorithms for incapacitated serial systems. Moreover,

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Fordyce and Webster (1984), Evans (1985), Chyr (1990, 1993), and Federgruen and Tzur (1991) presented some efficient algorithms to improve the computational efficiency. Meanwhile, due to the limitations of computational technology, several heuristic algorithms were investigated and developed by Gorham (1968), DeMatteis (1968), Silver and Meal (1973), and Groff (1979). Thus, most academic literatures emphasized to find the optimal solution with efficient algorithms or to get approximated solutions by adopting heuristic algorithms in requirements planning systems. However, there are few literatures to discuss how the behavior of all the possible solutions with associated total costs in DLS model. This study illustrates the characteristics of total costs for all the possible solutions in DLS model through the binary formulation framework.

After providing the introduction of motivations, the remainder of this study is organized as follows. Brief reviews of relevant literature about heuristics and excellent reviews are presented in section 2. The binary formulation framework and the trend of the total cost curve in DLS model is proposed in section 3. The sensitivity of the various algorithms are analyzed and recognized in section 4 and section 5. Finally, section 6 summaries the significant results of this study and some implications for future directions.

2. Literature Review

DLS model is subjected to vast interest due to the problem arises in many practical situations. Kaimann (1969) made the initial investigation aimed at identifying when to switch from the traditional EOQ model to the dynamic programming model. Some of the demand patterns with a well-defined termination point were used to compare the EOQ with the W-W algorithm. Kaimann (1969) also pointed out that the easy of use EOQ technique would outweigh the saving generated by using the other dynamic programming model even if there may be a small variation in total cost one way or other. Berry (1972) continued using the demand patterns to present a framework and to guide the production manager in selecting a procedure with respect to two criteria: inventory related costs, and computing time. Silver-Meal (1973) developed a simple heuristic for coping with the problem of selecting replenishment quantities under the time-varying demand patterns. Groff (1979) used the demand patterns to evaluate a major strength of part-period balancing. This study uses the demand patterns to recognize the sensitivity analysis and the variation of sensitivity interval in the DLS model.

A number of studies have provided an excellent review and comparison of solution approaches into lot sizing literature. Axsater (1985) derived the worst case performance bounds for a class of lot sizing heuristics and dealt with classical DLS problems without backlogging and capacity constraints. Zoller and Robrade (1988) explored numerous heuristics and indicated users of pertinent standard software benefit substantially from an incorporation of more recently proposed methods. Nydick and Weiss (1989) compared ten lot-sizing techniques using a set of widely varying parameters. Each technique can be evaluated for a specific data set that most closely approximates reality.

Simpson (2001) demonstrated not only various published lot sizing rules vary in terms of cost performance but also possess distinct strengths and weakness with respect to sparse demand patterns, short versus longer planning horizons, and degree of nervousness. Jans and Degraeve (2007) surveyed and compared the various meta-heuristics and distinctive solution approaches. Their respective advantages and disadvantages gave insight into more powerful hybrid algorithms, and provided the general guidelines for computational experiments by several examples. The calculating results of all heuristic algorithms are approximate or equal to the best solutions in the literature review.

Through the numerical examples of the demand patterns, this study locates the solutions in DLS model by the following 3 algorithms, including Wagner-Whitin (W-W), Silver-Meal (S-M), and Part Period Balancing (PPB). The locations of the solutions will indicate the sensitivity of these algorithms in DLS problems.

3. Binary Formulation Framework for Dynamic Lot-Sizing Model

This study makes capital of the binary formulation framework to visualize the complicated DLS model. The production policy of each period has two choices, “set-up (Order)” or “no set-up (Null)”, in the calculating process. Table 1 shows the production policies of the 4-period demand and it is composed of 16 kinds of the production policies. Nonetheless, some significant development deserves to be mentioned, if the first period demand isn’t empty, production policies can be reduced by 8 kinds of the production policies. In order to make computer for “Order” or “Null” separately, the production policies can translate into binary representation. The zero-one form for the 4-period production policies are also shown in Table 1, each production policy consists of a set of {0, 1}, which 0 represents “Null” and 1 represents “Order”. The 4-period production policies are from 0000 to 1111; moreover, if the first period demand isn’t empty, the 4-period production policies are from 1000 to 1111. Base on binary formulation framework, the demand of n-period has 2^n production policies, and it could be reduced by 2^{n-1} production policies when the first period demand isn’t empty.

According to the binary formulation framework, the total costs of DLS model of any production policies could be calculated effortlessly. Let D_t denote the demand at period t, S_t denote the set-up cost at period t, P_t denote the production policy {0, 1} at period t, H_t denote the unit holding cost for period t-1 through period t, and k denote the span from the anterior production policy {1} to the current production policy {0}. The total costs $TC(T)$ of the DLS model can then be written as:

$$TC(T)=\sum_{t=1}^{T}\left[P_tS_t+(1-P_t)D_t\sum_{t=t+1}^{t+k}H_t\right] \qquad \text{where } P_t = 0 \text{ or } 1,\ t = 1, 2,\ldots, T. \qquad (1)$$

Table. 1 The Zero-one Form of the 4-period Demand

production policy	Zero-One	Binary Formulation	production policy	Zero-One	Binary Formulation
Order, Order, Order, Order	1111		Null, Order, Order, Order	0111	
Order, Order, Order, Null	1110	$2^{(4-1)} \sim 2^4-1$	Null, Order, Order, Null	0110	$2^0-1 \sim 2^4-1$
Order, Order, Null, Order	1101	↓	Null, Order, Null, Order	0101	↓
Order, Order, Null, Null ⇨	1100 ⇨	$8_{10} \sim 15_{10}$	Null, Order, Null, Null ⇨	0100 ⇨	$0_{10} \sim 7_{10}$
Order, Null, Order, Order	1011	↓	Null, Null, Order, Order	0011	↓
Order, Null, Order, Null	1010	$1000_2 \sim 1111_2$	Null, Null, Order, Null	0010	$0000_2 \sim 0111_2$
Order, Null, Null, Order	1001		Null, Null, Null, Order	0001	
Order, Null, Null, Null	1000		Null, Null, Null, Null	0000	

4. Analysis of Dynamic Lot-Sizing Model

For base-case comparison, this study takes advantage of the example from Mekler (1993) to describe all scattered solutions for complicated DLS model. The total demand is 1200 and the schedule of 12-period demand is {10, 62, 12, 130, 154, 129, 88, 52, 124, 160, 238, 41}. This study follows four assumptions to make DLS model plainly. First, all of demands for each period must be available at the beginning of the period. Second, all of demands for a given period must be met and cannot be backordered. Third, the production set-up decisions are assumed to occur at the beginning of time intervals with zero lead time. Finally, the all demands are satisfied from inventory at the beginning of each period.

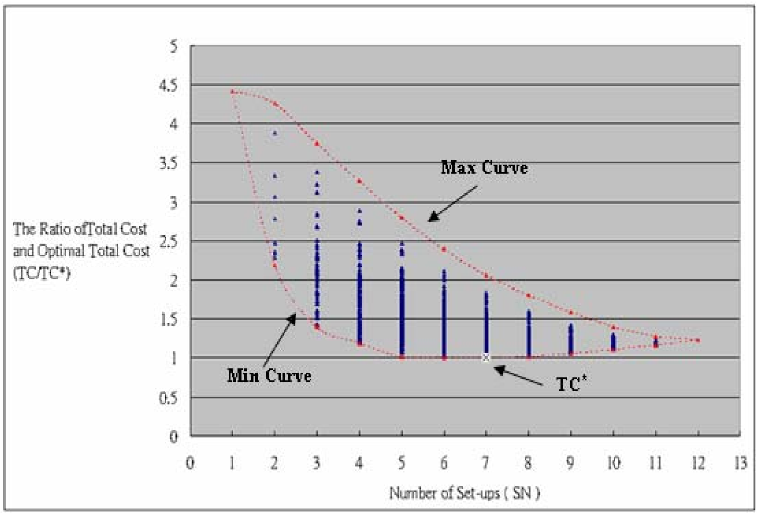

Fig. 1 Solutions Scatteration of All Production Policies with S_t = 300

From the binary formulation framework, the first period demand of the schedule isn't empty; hence the calculating results can be extended 2048 kinds of production policies. Furthermore, the inventory holding cost is 2 per unit of inventory per period through this study. The total costs of all production policies can be computed through the formulation (1), and the results are represented as the total costs TC divided by the optimum solutions TC^*, TC/TC^*. In the Fig. 1, the value of TC/TC^* in terms of percentage are plotted with respect to the number of set-ups (SN), wherein the set up cost is equal to 300. The best and worst solutions of every possible set-ups are plotted visibly both as minimum total cost curve and maximum total cost curve in the Fig. 1.

In order to recognize the scattered characteristics for all feasible solutions, this study inquires into set-up costs with 100 and 600 instead of 300 only for the initial example. The relations between TC/TC* and SN while the setup costs equal constants 100 and 600 are obviously shown in the Fig. 2 and the Fig. 3, respectively.

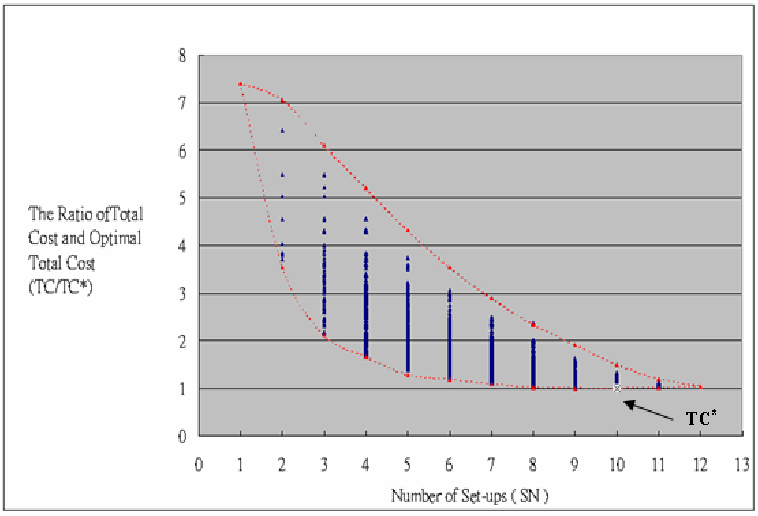

Fig. 2　Solutions Scatteration of All Production Policies with S_t = 100

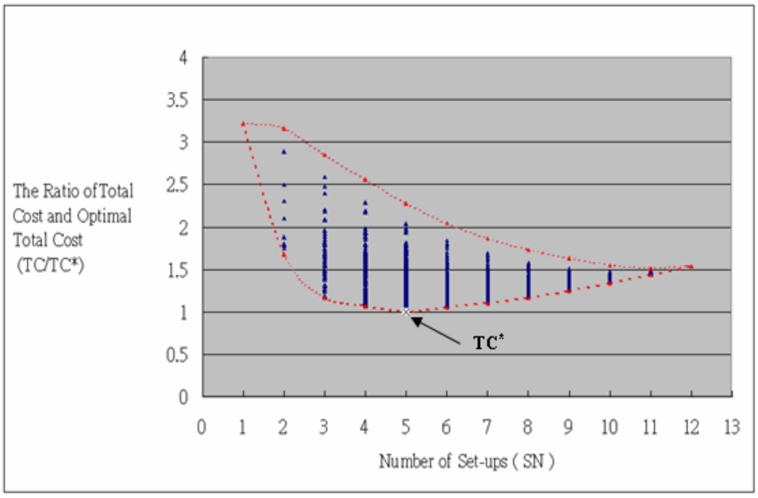

Fig. 3　Solutions Scatteration of All Production Policies with S_t = 600

From figures 1 to 3 the optimum SN of TC* will switch to right-hand side as the set-up cost decreases. The numbers of solutions within certain ranges of TC/TC* are indicated in Table 2. The relationship between feasible solutions and each SN consists of a fixed set {1, 11, 55, 165, 330, 462, 462, 330, 165, 55, 11, 1}. In the event, the widths of the feasible solutions will extend and the amount of the feasible solutions will increase when TC/TC* rises. The optimal solution is located at SN=5, SN=7 and SN=10 when the set-up cost with 600, 300, and 100. The result of calculation fits the aforesaid presentation that the optimum SN of TC^* will switch to right-hand side as the set-up cost decreases.

Table 2 Characteristics of Solutions Scatteration between TC/TC* and SN

SN / TC/TC*	1	2	3	4	5	6	7	8	9	10	11	12	Total	
	1	11	55	165	330	462	462	330	165	55	11	1	2048	100%
Set-up Cost = 100 ; Holding Cost = 2														
1.00-1.05	–	–	–	–	–	–	–	1	5	6*	3	1	16	0.78
1.05-1.10	–	–	–	–	–	–	1	9	11	7	2	–	30	1.46
1.10-1.20	–	–	–	–	–	3	19	7	40	18	3	–	90	4.39
1.20-1.30	–	–	–	–	1	15	55	115	57	24	2	–	269	13.13
1.30-2.00	–	–	–	28	159	334	355	195	52	–	1	–	1124	54.88
2.00-3.00	–	–	21	101	155	108	32	3	–	–	–	–	420	20.51
3.00-4.00	–	4	26	31	14	2	–	–	–	–	–	–	77	3.76
3.00-4.50	–	2	2	3	1	–	–	–	–	–	–	–	8	0.39
Set-up Cost = 300 ; Holding Cost = 2														
1.00-1.05	–	–	–	–	1	6	12*	5	1	–	–	–	25	1.22
1.05-1.10	–	–	–	–	4	16	29	31	9	1	–	–	90	4.39
1.10-1.20	–	–	–	3	29	94	140	132	84	31	5	–	518	25.29
1.20-1.30	–	–	–	12	56	112	121	94	54	22	6	1	478	23.34
1.30-2.00	–	–	25	117	224	228	159	68	17	1	–	–	839	40.97
2.00-3.00	–	7	26	32	16	6	1	–	–	–	–	–	88	4.30
3.00-4.00	–	3	4	1	–	–	–	–	–	–	–	–	8	0.39
3.00-4.50	1	1	–	–	–	–	–	–	–	–	–	–	2	0.10
Set-up Cost = 600 ; Holding Cost = 2														
1.00-1.05	–	–	–	–	4*	1	–	–	–	–	–	–	5	0.24
1.05-1.10	–	–	–	6	14	17	–	–	–	–	–	–	37	1.81
1.10-1.20	–	–	3	19	68	109	93	11	–	–	–	–	303	14.79
1.20-1.30	–	–	3	29	74	143	187	159	29	–	–	–	624	30.47
1.30-2.00	–	6	41	106	168	191	182	160	136	55	11	1	1057	51.61
2.00-3.00	–	4	8	5	2	1	–	–	–	–	–	–	20	0.98
3.00-4.00	1	1	–	–	–	–	–	–	–	–	–	–	2	0.10
3.00-4.50	–	–	–	–	–	–	–	–	–	–	–	–	0	0.00

In addition, the densities of the feasible solutions are taken on a tendency towards tight squeeze. Within the widths of SN and under the constant TC/TC^*, the densities denote the feasible solutions divide by widths of SN and center around the optimum TC^*. As $S_t = 100$, $H_t = 2$, and $TC/TC^* = 1.05$, the set of densities is {6/1, (6+5+3)/3, (6+5+3+1+1)/5}. The densities of the feasible solutions will diminish when the widths of SN increases and they will remove as well as the optimum SN of minimum TC/TC^*. The density characteristics of all the solutions in the DLS model reveal a tendency towards tight squeeze.

5. The Comparisons of Heuristics Algorithms

In this section, some algorithms that include W-W, S-M, PPB, are used to reflect the sensitivity interval in the DLS model. The sensitivity interval of DLS model in this study could be treated as the range between the calculating results TC/TC^* of the feasible solutions and the optimum SN of minimum TC/TC^*. Several arbitrary patterns that under the total demand 1200 are used to reflect the sensitivity of the algorithms in the DLS model. Table 3 shows the results of arbitrary demand patterns. Other than TC/TC^* as the only index in previous section, Ranking is an extra index used in Table 3. The value of Ranking indicates the rank of a solution out of all the solutions, for example, 1/66 represents the solution is the top rank solution within 66 dense ranking solutions. The result appears that all the approximated solutions for each algorithm in this study are close to DLS best solutions. This phenomenon may result from the tight squeeze density characteristics of all the solutions in the DLS model.

Table 3　The Results of Numerical Examples

Demand Pattern {100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100}						
Algorithms	St = 100, Ht = 2			St = 150, Ht = 2		
	SN	TC/TC*	Ranking	SN	TC/TC*	Ranking
W-W	12	1.000	1/53	12	1.000	1/66
S-M	12	1.000	1/53	12	1.000	1/66
PPB	12	1.000	1/53	12	1.000	1/66
	St = 200, Ht = 2			St = 250, Ht = 2		
W-W	6 ~ 12	1.000	1/26	6	1.000	1/62
S-M	6	1.000	1/26	6	1.000	1/62
PPB	12	1.000	1/26	6	1.000	1/62
	St = 300, Ht = 2			St = 600, Ht = 2		
W-W	6	1.000	1/41	4~6	1.000	1/25
S-M	6	1.000	1/41	4	1.000	1/25
PPB	6	1.000	1/41	6	1.000	1/25

Demand Pattern {80, 110, 135, 110, 55, 55, 110, 135, 135, 110, 55, 110}						
Algorithms	St = 100, Ht = 2			St = 150, Ht = 2		
	SN	TC/TC*	Ranking	SN	TC/TC*	Ranking
W-W	12	1.000	1/374	10	1.000	1/376
S-M	12	1.000	1/374	10	1.000	1/376
PPB	12	1.000	1/374	11	1.014	2/376
	S = 200, H = 2			S = 250, H = 2		
W-W	10	1.000	1/355	8	1.000	1/331
S-M	10	1.000	1/355	6	1.041	13/331
PPB	10	1.000	1/355	8	1.000	1/331
	St = 300, Ht = 2			St = 600, Ht = 2		
W-W	7	1.000	1/315	5	1.000	1/239
S-M	6	1.007	2/315	5	1.000	1/239
PPB	7	1.014	4/315	5	1.019	5/239

Table 3 The Results of Numerical Examples (Continue)

Demand Pattern {55, 90, 190, 90, 0, 0, 195, 55, 120, 110, 195, 100}						
Algorithms	St = 100, Ht = 2			St = 150, Ht = 2		
	SN	TC/TC*	Ranking	SN	TC/TC*	Ranking
W-W	10	1.000	1/445	9	1.000	1/431
S-M	10	1.000	1/445	9	1.000	1/431
PPB	10	1.000	1/445	10	1.015	3/431
	St = 200, Ht = 2			St = 250, Ht = 2		
W-W	6~7	1.000	1/379	5	1.000	1/403
S-M	6	1.000	1/379	9	1.087	21/403
PPB	9	1.036	6/379	8	1.084	20/403
	St = 300, Ht = 2			St = 600, Ht = 2		
W-W	5	1.000	1/399	5	1.000	1/364
S-M	6	1.387	113/399	5	1.326	118/364
PPB	7	1.061	9/399	5	1.000	1/364

Demand Pattern {10, 10, 15, 25, 75, 195, 270, 295, 250, 45, 0, 10}						
Algorithms	St = 100, Ht = 2			St = 150, Ht = 2		
	SN	TC/TC*	Ranking	SN	TC/TC*	Ranking
W-W	8	1.000	1/519	7	1.000	1/516
S-M	7	1.009	3/519	6	1.004	2/516
PPB	8	1.005	2/519	7	1.020	4/516
	St = 200, Ht = 2			St = 250, Ht = 2		
W-W	6	1.000	1/521	6	1.000	1/517
S-M	6	1.000	1/521	6	1.000	1/517
PPB	7	1.032	4/521	7	1.045	4/517
	St = 300, Ht = 2			St = 600, Ht = 2		
W-W	6	1.000	1/516	4	1.000	1/522
S-M	6	1.000	1/516	4	1.000	1/522
PPB	7	1.056	5/516	5	1.002	2/522

The results obtained from four arbitrary demand patterns and six types of set-up costs could be summarized as follows:

(1) W-W, S-M, and PPB—in that relationship between TC/TC^* and SN well in term of maximum observations: $TC/TC^* \leq 1.387$, and most results in this study reveal $TC/TC^* = 1.000$ throughout all examples.

(2) Ranking were sorted the place of TC/TC^* which cancel the repetition value; these algorithms are all most approximate the best rank of position.

(3) TC/TC^* has a significant impact on ranking that base on the algorithms comparing through the same demand patterns and type of set-up cost.

(4) The SN of the algorithms is close to the SN of DLS best solutions, and the SN reflects the sensitivity interval when the set-up costs adjust through the algorithms comparing.

(5) If n-period demands are average and similar in the patterns, like pattern 1, the calculating results of the algorithms will present no diversity of TC/TC^*.

(6) The result of TC/TC^* is analogous with the analysis from Silver and Meal's article, and the total costs of all algorithms are close to DLS best solutions.

The various algorithms reflect the limited sensitivity interval not only single example that include the discussion about original case , but also the arbitrary patterns and the six type of set-up cost by the numerical results. The results can be examined and observed through more comprehensive and arbitrary patterns in the future.

6. Conclusions

In this study we have presented a binary formulation framework to embody in dynamic lot-size problem, the TC/TC^* of all production policies can be calculated through a simple formulation. The selecting replenishments quantities under conditions of deterministic demand patterns where replenishments are restricted to the beginnings of discrete periods are examined and investigated through this study. Furthermore, we use several indices and various algorithms to recognize the sensitivity analysis between TC/TC^* and SN. The results of numerical examples reveal the major findings of this study are as follows:

(1) The various algorithms reflect the limited sensitivity interval through all examples. Moreover, the approximate value of rounding DSL best solution is perfectly acceptable.

(2) The SN of the algorithms reflects the sensitivity interval when the set-up costs adjust through the algorithms comparing. The experimental results also reveal the DLS minimum total costs curve is relatively flat near the best solution, especially the right side of the best one.

(3) The calculating results of TC/TC^* are analogous to past articles and the total costs of all algorithms are close to DLS best solutions. Moreover, TC/TC^* has a significant impact on ranking through all examples.

(4) The calculating results of algorithms and the characteristic of scattered solutions in the widths of SN reveal a tight squeeze for densities of solutions. The tight squeeze for densities of solutions will follow the shift of the optimum SN.

The significant implication in this study is DLS model has the sensitivity interval to reflect the calculating results of algorithms. The binary formulation framework is useful to represent the sensitivity interval in DLS model. Moreover, it could assist managers to examine the suitable range TC/TC^* and the calculating results of heuristic algorithms. If the calculating results of heuristic algorithms are within the range, the heuristic algorithms are appropriate for the operation situation. The analysis is directed toward improving the manager’s ability to make better decisions with regard to the total costs in choosing an appropriate algorithm for a requirements planning system.

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