Singling and Network Externality

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Abstract

In recent years, many researches discuss about economic behavior in the network. Our research focused on the effects of network externalities on the seller's signals of costs. We developed a game theoretical model, based upon incomplete and asymmetric information, to analyze such effects. Sequential Equilibrium (SE) and Pooling Equilibrium (PE) were then utilized to discuss the effects of network externalities given three propositions. The results indicate the network externalities have signal enhancing or inhibiting effects.

Keywords: Signaling, Network Externalities, Sequential Equilibrium, Pooling Equilibrium

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1. Introduction

In recent years, economic behavior and decision are related to network. Many economists and studies focus on the "network effect" or "network externalities". For example, there are many products having little or no value if they do not combined with others, just like home audio or video components and programming, which together provide entertainment services; automobiles, repair parts and service, which together provide transportation services; automatic teller machines and ATM cards, which together provide transaction services. These are all examples of products that are strongly complementary, although they need not be consumed in fixed proportions. Many economic processes can be regarded in terms of networks, in which combining a number of elementary inputs produce one or more goods or services. There are many reasons motivating us understand about economic behavior and decisions related to such network markets (see Katz and Shapiro 1994 and Economies 1996). First, the complexity of real-world network systems requires understanding of the conditional net effects of countervailing forces such as competition and integration. The duality between price competition for complementary products and quantity competition for substitutes can serve as a starting point here (see for this duality Singh and Vives 1984). Second, imperfect competition is a common phenomenon in many network systems and leads to complex interaction between market and network externalities (see, among others, Katz and Shapiro 1985).

The early development of the telephone service supplies a near-perfect example of a monopoly over a service having network externalities. Various users join a system that allows them to exchange messages with one another. Joining such a network is valuable precisely because many other households and businesses obtain components of the overall system. Because the value of membership to one user is positively affected when another user joins and enlarges the network. A user derives value from a communication network in rough proportion to the total number of subscribers. The telephone system in the U.S. was a monopoly based on Bell's 1876 patents over the basic technology.

In the market, firms always show their cost by introductory price. We can define the introductory price as a signal of cost. For example, online information services offer a more recent illustration of introductory pricing. First *CompuServe,* and later *Prodigy,* was introduced with a small sign-up charge and a low monthly fee. As the customer base grew, the services raised prices gradually. Users need not be connected by a physical network to realize network externalities. Users of computer operating systems and some general-purpose application packages receive indirect externalities as complementary hardware and software products become available. These establish the fact that introductory pricing may be an equilibrium pricing strategy when network externalities prevail and monopoly power is present.

The strategic decision of firms that face rational consumers is a lot of research has focused on in recent years. Especially, many economists have investigated the conditions under which an introductory or cyclical sale might be part of an optimal pricing strategy. There are few points have been identified. First, if consumers have incomplete information about product quality, then an introductory sale may be used to signal a high level of quality (Bagwell, 1985; Milgrom and Roberts, 1986). A separate literature links sales to consumer heterogeneity. Here, a sale emerges as a means of discriminating among heterogeneous consumers. Consumers may differ in their information (Varian, 1980), search characteristics (Salop, 1977; Shilony, 1977), or reservation prices (Conlisk, Gerstner, and Sobel, 1984). Finally, some research has suggested that durable (Conlisk *et al.*, 1984; Irvine, 1981; Sobel, 1984) or storable (Salop and Stiglitz, 1984) products are especially likely to be offered at sale prices.

Second, an incomplete cost information happened when a firm knows whether its production costs, but consumers do not observe costs. While not completely informed, consumers are rational and they therefore correctly anticipate that a low (high)-cost firm will charge a low (high) price in the second period.

And the equilibrium will take one of two forms. In a separating equilibrium, the introductory price selected by a firm depends on the firm's cost type. Thus, introductory price perfectly signals cost in a separating equilibrium. A pooling equilibrium is said to exist if the firm's introductory price is independent of its cost type. In a pooling equilibrium, introductory price provides no information.

In this paper, we discuss about the relation between signals and network externalities. Seller always shows the cost by introductory prices. We believe that this signal will be affected by network externalities. We want to know that how a monopolist prices a durable goods in different situations; besides, we want to know how equilibrium price moves according to our assumptions.

We build a simple game model in section 2. Assume that is a two-period model, the seller behaves as a monopolist. The product is durable, and the seller and buyer know the cost of each other. A Nash Bertrand equilibrium in symmetric information is therefore established. In section 3, we assume a game of incomplete information; that is, the cost of consumer is known but the cost of seller is private. Section 4 assumes that seller as a monopolist. We discuss the equilibrium situation and the seller's profit using a simple equation. In section 5, we discuss sequential equilibrium (SE) and pooling equilibrium (PE). We want to know how a buyer chooses under different strategic situations. Then, we study both equilibriums (SE and PE) using three propositions in the same section. Conclusions are presented in the final section.

2. Symmetric Information Nash Bertrand Equilibria

We consider a two-period game. Periods are denoted by 1 and 2 and firm is the domestic seller (s) and the consumer is the domestic buyer (b). The product we consider is durable. The seller behaves as a monopolist in the home market. The seller maximizes expected profits by posting introductory prices simultaneously in each period as the buyer makes the purchase. That is, the seller posts its decision on the choice of its prices in the market at the same time as the buyer does in the market. The seller is unable to set prices based on the buyer's types. The buyer can be of two types, high-valuation buyer (type H) and low-valuation buyer (type L). In period one, firm chooses an action. The seller's constant marginal cost is common knowledge. The seller faces symmetric demand curves in the home market in each period of the form

$$q_i = q_i(p_i, p_j) \qquad \text{for } i,j = s,b \text{ and } i \neq j \tag{1}$$

where $\partial q_i / \partial p_i < 0$ and $\partial q_i / \partial p_j > 0$. That is, the goods are substitutes and strategic complements. Given the price decision of the seller and the optimal responses of the buyers, the seller's profits in each period are denoted by

$$\pi_i = (p_i - c_i) q_i(p_i, p_j) \qquad \text{for } i,j = s,b, \text{and } i \neq j. \tag{2}$$

Maximizing π_i with respect to p_i for i= s, b yields a pair of first order conditions. If the c_i were common knowledge, the best response functions for the seller would be implicit in the first order conditions. They would generate a unique Nash-Bertrand equilibrium. Since the game is finite, the equilibrium would persist by backward induction for both periods. Of course, which equilibrium emerges depends upon whether the seller is commonly known to be high or low cost. If the seller is high cost, the equilibrium is given by the pair ($\overline{P}_s^H, \overline{p}_b^H$) , which is equal to $(-q_s^H \partial p_s^H / \partial q_s^H + c_s^H, -q_b^H + c_b^H)$. If it is low cost, the equilibrium is $(\overline{P}_s^L, \overline{p}_b^L)$, which equals to $(-q_s^L \partial p_s^L / \partial q_s^L + c_s^L, -q_b^L + c_b)$. Substituting the high cost and low cost equilibrium into the profit function, the profits for both equilibrium can be generated as $(\overline{\pi}_s^H, \overline{\pi}_b^H)$ and $(\overline{\pi}_s^L, \overline{\pi}_b^L)$.

3. Asymmetric Information about Cost

Next, we assume a game of incomplete information; that is, the seller has private information about its cost. The buyer's marginal cost, c_b, is common knowledge, but the seller's marginal cost, c_s, is known only to the seller. Suppose that for the seller's marginal cost, c_s, can *a priori* take two value: c_s^L (with the probability x) and c_s^H (with probability 1-x), where $c_s^L < c_s^H$. From the perspective of the risk-neutral buyer, the seller's expected marginal cost of production is

$$c_s^e = xc_s^L + (1-x)c_s^H \tag{3}$$

From previous argument, the equation $p_s = -q_s \partial q_s / \partial p_s + c_s$ shows that p_s is a non-decreasing function of the seller's marginal cost, given the price it believes the buyer will post. Thus, from the point of view of the buyer, who calculates the seller's expected price in the market from its *a priori* knowledge of the seller marginal costs, the first period expected prices is

$$p_s^e = xp_s^L + (1-x)p_s^H \tag{4}$$

Taking these values from (3), substituting into (4) yields:

$$P_s^e = -xq_s \partial p_s^L / \partial q_s - (1-x)q_s \partial p_s^H / \partial q_s + c_s^e \tag{5}$$

Recall the buyer is risk neutral, and it maximizes its expected profit with respect to p_d, such as

$$\underset{p_b}{Max}\, \pi_b = x(p_b - c_b)q_b(p_b, p_s^L) + (1-x)q_b(p_b, p_s^H) \tag{6}$$

The first order condition yields an analogue of (5):

$$p_b = -q_b \partial p_b / \partial q_b + c_b \tag{7}$$

Equation (5) and (7) can be solved to yield prices p_b^* and p_b^{e*}. Substituting p_b^* into the equation $p_s = -q_s \partial q_s / \partial p_s + c_s$ generated the asymmetric information equilibrium given by the pair $(\underline{p}_s^r, p_b^*)$ for r= L.H and the profit are defined as $\underline{\pi}_s^r(\underline{\pi}_b^r)$ for the seller (the customer). Note that the best response of a high (low) cost the seller to p_b^* is not the NB complete information equilibrium price $\overline{p}_s^L(\overline{p}_s^H)$. As figure 1 shows, the seller's best response (*BR*) is contingent on its cost, and it shifts to the right when the cost increase. Under asymmetric information, the seller would have an average *BR* curve (BR^e). Prices p_b^* and P_s^e are given by the intersection between the seller's *BR* curve and the buyer's average *BR* curve. The equilibrium price for the high (low) cost realization is lesser (greater) than the equilibrium price of their respective NB equilibria. Asymmetric information benefits (harms) the low (high) cost seller.

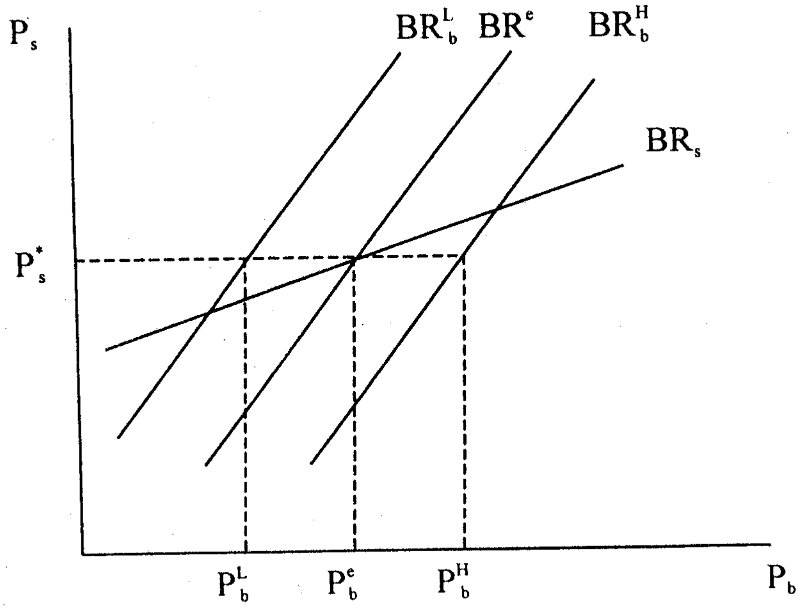

Figure 1. The Best Response

4. The Seller as a Monopolist

Assume that the seller faces a demand function that is not too convex in the market. Together with its constant marginal cost, this generates a concave profit function with sufficient first order conditions. Maximizing with respect to $\tilde{p}_b$ (where variables pertaining to the seller's market are denoted by a tilde) allow for determination of the monopoly prices in the domestic market. $\tilde{p}_{bm}^{H}(\tilde{p}_{bm}^{L})$ is termed for the high (low) cost realization of the seller. As the monopoly prices is an increasing function of the marginal cost, thus $\tilde{p}_{bm}^{H} > \tilde{p}_{bm}^{L}$. The profits corresponding to the prices are termed as π_M^H and π_M^L, respectively.

Since the purpose of the introductory pricing by the seller is to convince the buyer that it has low costs so as to raise earlier buyer's expectations about the likelihood of future sales, it is necessary to consider the seller as a monopolist in the domestic market. Using construction of a separating equilibrium (SE), we can derive conditions such that at equilibrium and after a sale in period one the monopolist optimally sells to both type of buyer in period two if it has low cost, but only to the high valuation buyer if it has high cost. Given that there are network externalities, a buyer's expected utility in the first period is higher the more he or she believes the seller to be a low-cost firm. This will result in an incentive for high-cost seller to masquerade as low-cost sellers in the first period. Finally, to distinguish itself from a high-cost seller, the low-cost seller has to set a very low price in the first period. Since, given the buyer's types, the seller's price is a function of its cost, this require that

$$d(\pi_M(c_s) - \pi_s(c_s, p_b))/dc_s < 0 \tag{8}$$

which can be derived using the envelop theorem.

5. Sequential Equilibrium and Pooling Equilibrium

The sequential equilibrium (SE) concept introduced by Kreps and Wilson (1982) is defined as a system of beliefs and a vector of strategy such that (Kyle, 1987):

(1) strategies are sequentially rational in that each player's strategy maximizes its expected payoff, given its beliefs and the equilibrium strategies of the other players, and

(2) beliefs are consistent in that on the equilibrium path, they are computed using Bayes' rule.

In separating equilibrium, Bayesian updating of the buyer's beliefs as to the seller's cost will be invoked. It is assumed observing $\underline{p}_s^L$ in a separating equilibrium in the first period induces exit. That is, the buyer will not pay the visit to the store if it knows that the seller has low costs.

A pooling Equilibrium (PE) is said to exist if the seller's introductory price is independent of its cost type. In a pooling equilibrium, introductory price provides no information; that is, the high-cost seller charges the same price as a low-cost seller in the first period. The high–cost seller offers its product at a reduced price, thereby temporarily halting the growth of its reputation as a high-cost monopoly. The resulting revenue from mimicking the low-cost realization can be used to offset relative losses incurred in the first period due to sale. The assumption is that if the buyer visits in first period, it is more profitable for the high-cost firm to charge p_s^L in the first period and have the buyer return in the second period than to charge P_s^H and have the buyer not return in period two. The second assumption is that visiting is worthwhile if no information has been received to the contrary. Suppose the seller chooses a first period price (${}^1p$) and second period price (${}^2p$). ${}^1p(c^i)$ is defined as the first period price that a monopolist of cost-type i would charge. Thus, if ${}^1p(c^H)={}^1p(c^L)$ so that $q(c=c^H|{}^1p(c^H))=\delta$, the buyer who visited in the first period will visit again in the second period. Second period visitation to follows the first period pooling under the condition, $\delta\pi_b(p_d^H)+(1-\delta)\pi(P_d^L)>0$.

Signal enhancing network externality

Proposition I: Network externality that raises the cost of mimicking a low cost realization of the seller, such that a first period partial equilibrium (PE) is converted to a SE benefits the seller when its expected profits are negative in a PE by precluding its visit to the seller's store when the seller has high costs.

Suppose that there is a PE in the absence of network externality, and that the buyer will not pay a visit to the store in this circumstance. That is expressed in (9) below, indicating that the buyer is unprofitable. The negative profits constitute injury to the buyer. Since wither cost-type realization is willing to signal low costs, the incompleteness of the buyer's information is not resolved. Network externality affecting the signaling costs of each realization of the seller may induce separation, thereby enabling the buyer to engage in Bayesian updating. Without the network externality, signaling is costless to the low cost realization. This is because $\underline{p}_s^L$ is the best response to p_d^* in period one, and $\tilde{p}_{sm}^L$ is the profit maximization price in the domestic market. With the network externality, a cost of $\tilde{\pi}_M^L-\tilde{\pi}_M^L(\underline{p}_s^L)$ can occur in the presence of the spillover effect. If it does not reveal, it must conceal its type by mimicking a high cost seller in posting p_s^H in period one. This entails the buyer's profit of $\underline{\pi}_b^L(\underline{p}_b^H,P_s^*)$ and a cost of $\underline{\pi}_b^L-\underline{\pi}(\underline{p}_b^H,p_d^*)$. If it reveals its low costs, it becomes a monopolist in the second period, earning $\delta\pi_M^L$, where the discount rate is δ. Concealing its cost in the first period by mimicking the high cost realization induces a best response to p_d^* in period two of p_b^L. This generates profits of $\delta\pi_s^L$, and the gain from cost revelation is $\delta(\pi_M^L-\underline{\pi}_s^L)$. If the cost of revealing its type is less than the gain to second period monopolization in the presence of a network externality, then (10) below must hold.

The high cost seller incurs a signaling cost in absence of network externality. Since the optimal response to p_d^* is $\underline{p}_s^H$, posting $\underline{p}_s^L$ sacrifices the market profits in period one. As the buyer cannot observe the price posted by the seller, the high cost seller can maximize profits by posting $\tilde{p}_{sm}^H$. This includes profits of $\underline{\pi}_s^H(\underline{p}_s^L, p_d^*)$ for the high cost (type H) seller, and a cost of signaling of $\underline{\pi}_s^H - \underline{\pi}_s^H(\underline{p}_s^L, p_d^*)$. The gain from second period monopolization of the market is $\delta(\pi_M^H - \underline{\pi}_s^H)$. The second period gain is relative to the high cost NB equilibrium, as the low cost firm will reveal itself. For the type H seller to be willing to mimic the low cost firm (type L), (11) below must hold.

$$\pi_b^* = x\underline{\pi}_b^L + (1-x)\underline{\pi}_b^H < 0 \qquad \text{where } \underline{\pi}_b^L < 0 \text{ and } \underline{\pi}_b^H \geq 0 \tag{9}$$

$$\tilde{\pi}_M^L - \tilde{\pi}_M^L(\underline{p}_s^L) + \underline{\pi}_s^L(\underline{p}_s^L) + \underline{\pi}_s^L(\underline{p}_s^H, p_d^*) - \underline{\pi}_s^L < \delta(\pi_M^L - \underline{\pi}_s^L) \tag{10}$$

$$\underline{\pi}_s^H - \underline{\pi}_s^H(\underline{p}_s^L, p_d^*) < \delta(\pi_M^H - \overline{\pi}_s^H) \tag{11}$$

Signal jamming network externality

Proposition II: Network externality that raises the cost of revealing its type for a low cost realization of a seller and induces it to mimic a high cost realization, such that a first period SE is converted to a PE, harm the buyer when its expected profits are negative in a PE by inducing its visit to the store when the seller has high costs.

Suppose that (9) continues to hold, so that the buyer is unprofitable in a PE. Assume also that (11) is violated. Hence a SE occurs in the absence of network externality. The buyer does not visit (visit) if the seller has low (high) costs.

Assume that with network externality, the cost of revealing for a type L seller exceeds the benefit of the second period monopolization. Thus, the inequality in (11) is reversed. Since posting $\underline{p}_s^L$ is the best response to P_d^* for low cost seller, any other price is costly in terms of market profits. If it does bot post $\underline{p}_s^L$, the low cost seller must mimic high cost seller by posting $\underline{p}_s^H$. Furthermore, as $\underline{p}_s^H$ is its best response to P_d^*, the high cost seller can mimic the low cost seller. Since the buyer cannot observe the seller's price on the lack of network externality, the high cost seller can post $\tilde{p}_{sm}^H$ and incur a signaling cost of zero.

A PE at the pair $(\underline{p}_s^H, p_d^*)$ exists and the buyer is not able to engage in posterior updating of its beliefs. It expects negative profit in period two because of (10). Thus the buyer does not visit after period 1, even though it is profitable $((\underline{\pi}_b^H \geq 0)$.

The Buyer gains benefits

Proposition III: Network externality that raises the cost of revealing for a low cost realization of a seller and induces it to mimic a high cost realization, such that a first period SE is converted to a PE, benefits the buyer when its expected profits are positive in a PE by postponing its visit to the store when the seller has low costs.

Suppose that (9) does not hold, so that the buyer is profitable in a PE. Also assume that (10) and (11) are violated. As mentioned earlier, Bayseian updating does not take place because of the PE at the pair $(\underline{p}_s^H, p_d^*)$. However, the buyer expects positive profits in period two. This is despite the fact that the buyer would not visit if it knew that the seller was low cost. It will learn the seller's type in period two, as each realization plays its optimal reply to P_d^*. Since $\underline{\pi}_s^L < 0$, the buyer will incur a second period loss if the seller is low cost. Since the loss occurs in period one without network externality, the buyer gains by its postponement for one period.

6. Concluding Remarks

The purpose of this paper is to investigate the effects of network externalities on the sending of signals from the seller to the buyer. The present example concerned signals of production cost through prices. The network externalities can either enhance or jam the signal. It is not the intention of this paper to consider all possible producer equilibria within the assumed structure. Rather, it is to demonstrate that network externalities can have these effects. The effects are usually, but not always, beneficial to the buyer.

The paper also differs from traditional path in that buyer rationally use available information as a signal of price. In addition, out of equilibrium behavior and beliefs are dealt with explicitly. However, the scope of this paper is quite limited. Further research can be expanded to understand the role of signaling and price reputation in different settings of network effect.

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訊號與網路外部性

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摘要

在網路的世界中，我們常會去探討網際網路對於經濟行爲的影響，其中有許多研究著重於網路外部性 (network externalities)的影響。本研究探討網路外部性下獨占廠商該如何對購買者散佈其成本訊號。我們假設一個賽局模型來討論在資訊對稱以及資訊不對稱之下的奈許均衡解 (Nash Bertrandequilibrium)。再利用SequentialEquilibrium (SE)與Pooling Equilibrium (PE)兩種均衡模式來探討網路外部性對於成本訊號的幾種影響。結果發現網路外部性與成本訊號之間有增加 (enhancing)與阻礙(jamming)兩個關係存在。

關鍵詞：訊號，網路外部性，均衡，賽局理論