行政院國家科學委員會輔助專題研究計劃報告

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期望效用極大下的動態最適避險比率

Optimal Dynamic Hedge Ratios in an Expected-Utility-Maximization Model

中文摘要

二元序列相關條件異質變異模型所導出的動態避險比率，與傳統的最小平方法所導出的靜態避險比率，皆為變異極小的最適解，即假設決策者均有高度的風險趨避傾向，而忽略不同決策者不同的風險偏好，以及避險工具的操作仍有可能提高風險資產的收益。本研究透過決策目標為極大化期望效用的避險模型，發現最適避險比率與決策者的風險偏好程度不相關，其所導出的避險策略可增加避險者的期末財富；惟其風險高於其他兩種避險比率的結果。

關鍵詞：動態規畫，最適避險比率，期望效用極大

Abstract

The conventional static hedge ratios derived from OLS and the more recently developed dynamic hedge ratios from bivariate GARCH are two of the most widely discussed hedging strategies in literature. However, these two broadly used procedures are both based on the minimum-variance objective, which implicitly assumes that all hedgers are extremely risk averse. This assumption also ignores the potential benefits of the usage of risk-management instruments in the increase of asset returns. Through an expected-utility-maximization model, the present study provides numerical evidences that the optimal futures hedge ratios do not depend on the risk preferences. Using simulated price scenarios, the optimal dynamic hedge ratios from the proposed model have a better performance in the increase of final wealth.

Keywords: dynamic programming, optimal hedge ratio, expected-utility-maximization

INTRODUCTION

A majority of studies employed the bivariate general autoregressive conditional heteroskedastic (GARCH) models to derive the time-varying optimal hedge ratios, which successfully take into account the cond-ditioning information available when the hedge is placed (e.g., Myers, 1991, Bailie and Myers, 1991; Kroner and Sultan, 1993; Gagnon and Lypny, 1995; etc.). However, these two widely used procedures to optimal hedge ratios are based on the minimum-variance objective, which implicitly assumes that all hedgers are extremely risk averse and ignores the potential benefits in the increase of asset returns. The present study takes one more step forward to formally derive the “optimal” hedge ratio in an expected-utility-maximization framework, allowing various degrees of risk aversion, for a special case of storable commodities. The hedging performance for various degrees of risk aversion is investigated with comparison to the static OLS and dynamic GARCH results.

THOERETICAL MODEL

Assume that an agent involved in the processing of storable commodities expects to profit from the hedges against the price risk associated by trading in the futures markets. The agent does not sell his storage thus fixes his cash position until the last period but is allowed to revise his futures position at each decision node from initial period to final period, T, to optimize his object function defined as:

$$[1] \quad \max_{\{b_t\}} E_0[U(w_T)] \quad s.t.$$

$$[2] \quad w_{t+1} = (1+r)[w_t + (b_t - b_{t-1}) f_t]$$

$$[3] \quad w_T = w_{T-1} + s_0 p_T + (b_T - b_{T-1}) f_T$$

$$[4] \quad 0 \le b_t \le s_0$$

where w_t and b_t are, respectively, wealth levels and futures position at t, which are the state variables in the dynamic programming. s_0 is the amount of storage at the initial stage. $b_{t+1} - b_t$ denotes the futures contract bought (sold) at t if it is positive (negative). p_t and f_t are cash and futures prices at t, which are the stochastic state variables, depicting the risky environment encountered by the agent. r is a risk-free interest rate. [4] restricts the agents from pure speculation.

DYNAMIC PROGRAMMING METHOD

A great deal of efforts in this study has been devoted to constructing the stochastic space of the dynamic hedging decision model. This starts with specifying an econometric model, which describes the joint generating process of cash and futures prices. The analysis on the cash and futures price movements is similar to Park and Switzer (1995) in a bivariate GARCH specification with constant correlation coefficient (ñ) as showed in Table 1, but the hedging ratios is not retrieved directly from the conditional variance-covariance matrix. Instead, time-varying probability-based transition matrices are first constructed from the parameterized econometric model, which then become the inputs, as time-varying stochastic states, into the decision model. The expected-utility-maximization model is solved numerically in a discrete dynamic programming procedure with which the optimal hedge ratios can be calculated.

To simulate the joint distributions of cash and futures prices, we first create two random draws independently from the standard normal distribution, denoted as $X = [x_1, x_2]'$, and let M be a lower triangular transform matrix, defined as

$$[5] \qquad M = \begin{bmatrix} \sqrt{\hat{h}_{p,t}} & 0 \\ ñ\sqrt{\hat{h}_{f,t}} & \sqrt{(1-ñ^2)\hat{h}_{f,t}} \end{bmatrix}$$

where the “hat” represents the conditional variances converted through the previously estimated econometric model by iteration from the initial period. We then have MX = $[\hat{å}_p, \hat{å}_f]'$ and through the mean equation yields a set of cash and futures prices.

NUMERICAL RESTULS

Using the simulated price scenarios, the “optimal” hedge ratios from the expected-utility-maximization model are presented in table 2. We also calculate the static hedge ratio from the OLS method and dynamic hedge ratios directly from the Bivariate GARCH model. This time-varying hedge ratios can be expressed with the variance estimates: $\hat{h}_{pf,t} \big/ \hat{h}^2_{f,t} = \hat{ñ} \sqrt{\hat{h}^2_{p,t}} \big/ \sqrt{\hat{h}^2_{f,t}}$.

From Table 2, it appears that the level of hedge ratios suggested by the underlying research, in general, is much less than those

Table 1 Estimates of the Bivariate GARCH model

$$1000 \times (\ln p_t - \ln p_{t-1}) = \underset{(4.035)}{4.947}$$

$$h_{p,t} = \underset{(4.035)}{397.92} + \underset{(5.289)}{0.294}\,\varepsilon^2_{p,t-1} + \underset{(9.187)}{0.502}\,h_{p,t-1} - \underset{(0.12)}{5.206}\sin(2\pi\, n_t / 42) - \underset{(3.337)}{49.726}\cos(2\pi\, n_t / 42)$$

$$1000 \times (\ln f_t - \ln f_{t-1}) = \underset{(2.060)}{1.890}$$

$$h_{f,t} = \underset{(3.843)}{132.109} + \underset{(5.438)}{0.355}\,\varepsilon^2_{f,t-1} + \underset{(11.54)}{0.604}\,h_{f,t-1} + \underset{(5.185)}{301.848}\sin(2\pi\, n_t / 42) + \underset{(3.205)}{105.592}\cos(2\pi\, n_t / 42)$$

$$h_{pf,t} = \underset{(31.708)}{0.71}\sqrt{h_{p,t}}\,\sqrt{h_{f,t}}$$

Note: the estimation period is from the third Wednesday of October to the third Wednesday of July, 1985-997, for weekly cash and futures (July contract) prices for corn. $\sin(2\pi n/42)$ and $\cos(2\pi n/42)$ represent continuous seasonal factors at t, where n denotes the n-th Wednesday after the third week of October each year. The numbers in parentheses are the absolute values of the t-statistics.

from other approaches, especially during January and June. An interesting result is that the optimal hedge ratios across the utility functions and the degrees of risk aversions do *not* have significant differences. This implies that the optimal hedging strategies are independent from the hedger's risk preferences. We also found that the optimal hedging strategies derived from our expected-utility-maximization model leads to a higher final wealth, 30% more than the GARCH results on average, a potential benefit that has been ignored from other popular approaches. However, a tradeoff coupled with the better performance in the wealth level is identified, which is an increase in risks, about 66.76% more in standard deviation of final wealth than the GARCH results.

CONCLUSION (including self evaluation)

It has been concluded from our study that the optimal hedging strategies are independent from the hedger's risk preferences. The optimal hedging strategies proposed from our expected-utility-maximization model has the potential to increase the hedger's final wealth, though higher risks may in the meantime be involved.

However, we also recognized that the model we specified is encountered with a question of robustness since the resulting optimal strategies are very sensitive to the stochastic space we constructed. More experiments are required for a complete study.

This study has mostly followed the procedure first proposed, however, with one exception: the cash position in the decision-making model is not allowed to change over time. The main reason is that implementing the discrete stochastic dynamic programming becomes exponentially complicated and time consuming even with one choice variable

Table 2 Optimal Hedge Ratios for Expected-Utility-Maximization, Bivariate GARCH, and OLS, and Comparison of Hedge Effectiveness

	Nov	Dec	Jan	Feb	March	April	May	June	% of w_T increase	% s.d of w_T increase
CRRA										
0.00001	.4260	.4081	.2739	.1774	.1521	.2469	.2968	.3849	29.87	66.76
0.25	.4257	.4081	.2740	.1783	.1514	.2475	.2964	.3897	29.84	66.73
0.5	.4257	.4081	.2723	.1789	.1514	.2741	.2966	.4029	29.84	66.73
0.75	.4257	.4081	.2723	.1789	.1492	.2465	.2962	.4061	29.81	66.66
0.99999	.4257	.4081	.2715	.1789	.1480	.2462	.2962	.4061	29.80	66.66
CARA										
0.00001	.4257	.4081	.2715	.1789	.1480	.2462	.2962	.4061	29.80	66.73
0.25	.4257	.4081	.2740	.1790	.1515	.2475	.2964	.3934	29.83	66.66
0.5	.4263	.4081	.2740	.1763	.1527	.2457	.3019	.3764	30.32	66.60
0.75	.4266	.4081	.2694	.1739	.1519	.2414	.3016	.3722	30.30	66.48
0.99999	.4266	.4031	.2651	.1737	.1529	.2456	.3009	.3632	30.25	66.03
BGARCH	.5458	.6105	.8404	.7144	.6500	.7440	.7892	.8373		
OLS	.9108	.9108	.9108	.9108	.9108	.9108	.9108	.9108	5.55	1.75

Note: CRRA is the utility function of constant relative risk aversion, in the form of $U(w_T)= w_T^{â}$ and CARA is $U(w_T)$ =-exp[(â-1) w_T]. The coefficient of risk aversion is equal to 1-â, for 0 < â < 1. Therefore, when â = 0, the agent is assumed to be risk neutral whereas â = 1 is extreme risk averse. The "% of w_T increase" is calculated as $(w_T^{Others} - w_T^{GARCH}) / w_T^{GARCH}$. The "% s.d. (standard deviation) of w_T increase" is calculated as $(ó^{Others} - ó^{GARCH}) / ó^{GARCH}$.

(cash position) added which will further increase one more associated stochastic variable (cash price). Another reason is that the results derived from the two conventional approaches are based on the assumption of fixed cash position. In considering the consistency for comparison, we therefore impose the restriction on the cash position to be constant until the final period.

LITERATURE REVIEW

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